

## Calculus Challenge Examination 2008 – Commentary

- [9] 1. (a) Many students did not look at one-sided limits for this question. The most common reason given was “1/0” DNE so the limit DNE.  
(b) Well done.  
(c) Very few students knew the limit definition for  $e$ .
- [6] 2. Well done.
- [6] 3. Very well done.
- [6] 4. (a) Most students identified one possible  $f$ .  
(b) Overall well done question aside from notation abuse. For a number of students the equal sign slipped between the limit notation and its argument. Or the limit notation got dropped before completing the limit. Or equal signs were non-existent. A half mark to one full mark was taken off for notation abuse. About one third of the students used l’Hospital’s Rule.
- [8] 5. Only a handful of students earned full marks. Most students did not know what to do with this question and failed to see that the slope can be written in two ways and then equated.
- [5] 6. This was an interesting question. Some students used integration and FTC. Some students got all the way to  $\frac{d\theta}{dt} = \frac{1}{\sec^2 \theta}$  and then stopped. Some students tried using the definition but got nowhere mostly due to a misunderstanding of the inverse by claiming that  $\tan^{-1} t$  is equal to  $\frac{1}{\tan t}$ , or  $\frac{\sin^{-1} t}{\cos^{-1} t}$ , or other such nonsense. About a quarter of the students left this question blank.
- [6] 7. Mostly well done question except temptation was high for students to connect the graphical pieces of the derivative. It was nice to see that most students handled the corner and endpoints well.
- [6] 8. Very well done.
- [6] 9. A handful of students used the First Derivative Test to make conclusions about concavity indicating a lack of understanding. Most students approached this question correctly but the most common marks were 4 or 5 due to incorrect derivatives.
- [8] 10. The most common mark was 6.5. Most students approached this problem the way shown in the solution set. A handful of students set up the problem in terms of  $\theta$  but only one student achieved a perfect score. Some students even introduced the angle  $\beta$  in the right triangle and solved the problem correctly that way. The most common mistakes were leaving out the domain, concluding that  $V'(c) = 0$  means  $V(c)$  is the maximum, or interpreting  $\frac{dV}{dt}$  as the derivative of  $V$  with respect to  $h$ ,  $r$ , or  $\theta$ .
- [8] 11. This turned out to be an interesting question as well. Many students did well but there were a lot of misunderstandings: Some students had trouble setting up the distance formula. Some students thought the question was asking for  $dy/dt$ . Some students thought this problem was about a moving particle with  $x$  as the position function and  $v=x'$  and tried to find  $t$  assuming constant speed. Only a few students had no idea what this question was asking for.

- [9] 12. Aside from the fact that an error crept into this question where part (f) should have been as described below the students did very well. Most students ignored the ambiguity caused by the error earning full marks. About a quarter pointed out the ambiguity and also earned full marks. Only a very small minority of students did not know what to do with this question and judging from the remaining parts of their exams it seemed that this showed a lack of understanding of calculus overall.
- (f)  $f'(x) < 0$  for  $x < -3$ ,  $-3 < x < 1$ , and  $x > 4$  (should be  $x > 3$ ), and  $f'(x) > 0$  for  $1 < x < 4$  (should be  $1 < x < 3$ ).
- [9] 13. This turned out to be an interesting question. Students basically split into three groups: A fair number of students answered this question completely or almost correct. A small group of students answered part (a) correctly finding the average rates and comparing them but failed to get anywhere with parts (b) and (c). The core group of students failed to understand what part (a) was asking for but answered parts (b) and (c) correctly or almost correctly.
- [8] 14. The majority of students did well on this question. Some students missed graphing the line  $y = \pi$  thereby getting a different region but still earned up to 7 marks for this problem. Many students had  $dx$  missing in their integral without a mark penalty.