SFU - UBC – UNBC - UVic

Calculus Challenge Exam

June 8, 2006, 12 noon to 3 p.m.

Host: SIMON FRASER
**   UNIVERSITY**

Student signature

INSTRUCTIONS

1. Show all your work. Full marks are given only when the answer is correct, and is supported with a written derivation that is orderly, logical, and complete.

2. Calculators are optional, not required. Correct answer that is calculator ready, like $3 + \ln 7$ or e^2, are preferred.

3. Any calculator acceptable for the Provincial Examination in Principles of Mathematics 12 may be used.

4. A basic formula sheet has been provided. No other notes, books, or aids are allowed. In particular, all calculator memories must be empty when the exam begins.

5. If you need more space to solve a problem on page n, work on the back of the page n-1.

6. CAUTION – Candidates guilty of any of the following or similar practices shall be dismissed from the examination immediately and assigned a grade of 0:

    (a) Using any books, papers or memoranda.

    (b) Speaking or communicating with other candidates.

    (c) Exposing written papers to the view of other candidates.

Question	Maximum	Score
1	4	
2	10	
3	6	
4	9	
5	6	
6	15	
7	4	
8	6	
9	4	
10	8	
11	8	
12	6	
13	4	
14	4	
15	6	
Total	100	

[2] 1. (a) Sketch a rough graph of $y(x) = \dfrac{|x-2|}{x-2}$ near $x = 2$.
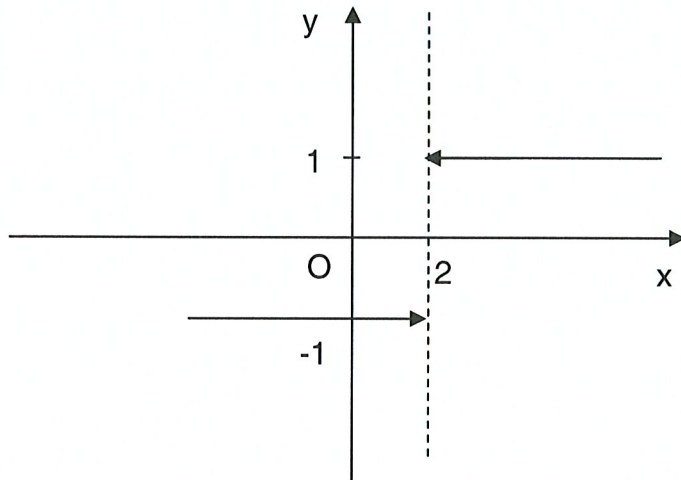
[2] (b) Evaluate $\displaystyle\lim_{x \to 2^-} \dfrac{|x-2|}{x-2}$.

ANSWER:

-1

JUSTIFY YOUR ANSWER

Note that

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$$

Then

$$\lim_{x \to 2^-} \dfrac{|x-2|}{x-2}$$

$$= \lim_{x \to 2^-} \dfrac{2-x}{x-2}$$

$$= \lim_{x \to 2^-} \left[-\left(\dfrac{x-2}{x-2} \right) \right]$$

$$= -1.$$

[6] 2. (a) Given that

$$f(x) = \frac{x^2 e^{4x}}{\sin x + \cos(2 - 3x)}$$

find $f'(x)$. No simplification is necessary.

ANSWER

$$f'(x) = \frac{\left[2xe^{4x} + 4x^2 e^{4x}\right]\left[\sin x + \cos(2 - 3x)\right] - x^2 e^{4x}\left[\cos x - (-3)\sin(2 - 3x)\right]}{\left[\sin x + \cos(2 - 3x)\right]^2}$$

[4]   (b) Suppose that functions $F(x)$ and $G(x)$ satisfy the following properties:

$$F(3) = 2, \quad G(3) = 4, \quad G(0) = 3,$$
$$F'(3) = -1, \quad G'(3) = 0, \quad G'(0) = 2.$$

If $T(x) = F(G(x))$ and $U(x) = \ln(F(x))$, find $T'(0)$ and $U'(3)$.

ANSWER: $T'(0) = -2$
$$U'(3) = -\frac{1}{2}$$

SHOW YOUR WORK

$T(x) = F(G(x))$

$\Rightarrow T'(x) = F'(G(x))G'(x)$

$\Rightarrow T'(0) = F'(G(0))G'(0)$

$\Rightarrow T'(0) = F'(3)(2)$

$\Rightarrow T'(0) = -2.$

$U(x) = \ln[F(x)]$

$\Rightarrow U'(x) = \dfrac{F'(x)}{F(x)}$

$\Rightarrow U'(3) = \dfrac{F'(3)}{F(3)}$

$\Rightarrow U'(3) = \dfrac{-1}{2}$

$\Rightarrow U'(3) = -\dfrac{1}{2}.$

[3] 3. (a) Let $f(x) = xe^x$. Express $f'(0)$ as a limit by using the definition of derivative and hence evaluate $f'(0)$.

ANSWER:

$$f'(0) = 1$$

SHOW YOUR WORK

$$
\begin{aligned}
f'(0) &= \lim_{h \to 0} \frac{f(h) - f(0)}{h} \\
&= \lim_{h \to 0} \frac{h(e^h) - 0(e^0)}{h} \\
&= \lim_{h \to 0} \frac{h(e^h)}{h} \\
&= \lim_{h \to 0} e^h \\
&= 1
\end{aligned}
$$

[3] (b) Let $g(x) = \left| 1 - x^2 \right|$. Does $g'(x)$ exist at $x = 1$?

ANSWER:

No

EXPLANATION

Function $g(x)$ is differentiable at $x = 1$ if $\lim_{h \to 0} \dfrac{g(1+h) - g(1)}{h}$ exists. Notice that

$$
g(x) = \begin{cases} -(1 - x^2), & x < -1 \text{ or } x > 1 \\ 1 - x^2, & |x| \le 1 \end{cases}
\qquad \Leftrightarrow \qquad
g(x) = \begin{cases} x^2 - 1, & x < -1 \text{ or } x > 1 \\ 1 - x^2, & |x| \le 1 \end{cases}
$$

Then

$$
\lim_{h \to 0^+} \frac{g(1+h) - g(1)}{h} = \lim_{h \to 0^+} \frac{[(1+h)^2 - 1] - [1^2 - 1]}{h} = \lim_{h \to 0^+} \frac{2h + h^2}{h} = \lim_{h \to 0^+} (2 + h) = 2,
$$

$$
\lim_{h \to 0^-} \frac{g(1+h) - g(1)}{h} = \lim_{h \to 0^-} \frac{[1 - (1+h)^2] - [1 - 1^2]}{h} = \lim_{h \to 0^-} \frac{-2h - h^2}{h} = \lim_{h \to 0^-} (-2 - h) = -2.
$$

The one-sided limits are not equal, hence, $\lim_{h \to 0} \dfrac{g(1+h) - g(1)}{h}$ does not exist.

Therefore, $g'(x)$ does not exist at $x = 1$.

[9] 4. An open steel cylinder is required which will hold V (cm^3) of liquid. The thickness of the walls and base of the cylinder is d (cm). Find the height h (cm) and inner radius r (cm) of the cylinder which minimize the amount of steel required. The diagram shows the cross-section of the cylinder through its axis. HINT: Express M, the volume of steel required, as a function of x.

ANSWER:
$$h = x = \sqrt[3]{\frac{V}{\pi}}$$

___

SHOW YOUR WORK
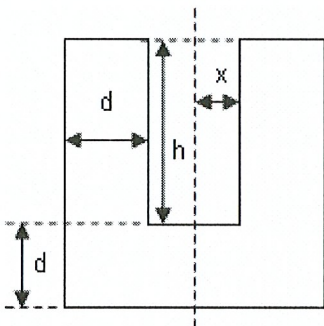
Volume of liquid that the container can hold is given by:

$$V = \pi x^2 h \implies h = \frac{V}{\pi x^2}.$$

Volume of steel required to build the cylinder is found as:

$$
\begin{aligned}
M &= \pi(x+d)^2(d+h) - \pi x^2 h \\
&= \pi d(x+d)^2 + \pi h[(x+d)^2 - x^2] \\
&= \pi d(x+d)^2 + \pi h[2xd + d^2] \\
&= \pi d(x+d)^2 + \pi\left[\frac{V}{\pi x^2}\right][2xd + d^2] \quad \text{(by substituting for } h\text{)} \\
&= \pi d(x+d)^2 + \left[\frac{V}{x^2}\right][2xd + d^2] \\
&= \pi d(x+d)^2 + \frac{2Vxd}{x^2} + \frac{Vd^2}{x^2} \\
&= \pi d(x+d)^2 + \frac{2Vd}{x} + \frac{Vd^2}{x^2}.
\end{aligned}
$$

Notice that $x > 0$, so M is well-defined. Compute the derivative of M with respect to x.

$$
\begin{aligned}
M'(x) &= 2\pi d(x+d) - \frac{2Vd}{x^2} - \frac{2Vd^2}{x^3} = \frac{2\pi d x^3(x+d) - 2Vd(x+d)}{x^3} \\
&= \frac{2d(\pi x^3 - V)(x+d)}{x^3}.
\end{aligned}
$$

Setting $M'(x) = 0$ implies $V = \pi x^3$ (as $x \neq -d$ since $x, d > 0$). (*)

Now investigate the critical point $x_m = \sqrt[3]{\dfrac{V}{\pi}}$ by calculating the second derivative of M.

$$M''(x) = 2\pi d + \frac{4Vd}{x^3} + \frac{6Vd^2}{x^4}.$$

Notice that $M''(x) > 0$ for all $x > 0$. Therefore, from the Second Derivative Test it follows that $M(x)$ reaches a global minimum at $x_m = \sqrt[3]{\dfrac{V}{\pi}}$.

Finally, $V = \pi x^2 h$ is given and $V = \pi x^3$ (from *). Hence, it follows that $h_m = x_m$.

5. A car moves in a straight line. At time t (measured in seconds) its position (measured in metres) is

$$s(t) = \frac{1}{100}t^3, \quad 0 \le t \le 5.$$

[2] (a) Find its average velocity between $t = 0$ and $t = 5$.

ANSWER:
$$\frac{1}{4}$$

JUSTIFY YOUR ANSWER

To find the average velocity take the distance travelled divided by the time elapsed:

$$V_{ave} = \frac{s(5) - s(0)}{5} = \frac{1}{100}\frac{5^3}{5} = \frac{1}{4}$$

[2] (b) Find its instantaneous velocity for $0 < t < 5$.

ANSWER:
$$\frac{3t^2}{100}$$

JUSTIFY YOUR ANSWER

To find instantaneous velocity take the derivative of the position function.

$$v = \frac{ds}{dt} = \frac{3t^2}{100}$$

[2] (c) At what time is the instantaneous velocity of the car equal to its average velocity?

ANSWER:
$$\frac{5\sqrt{3}}{3}$$

JUSTIFY YOUR ANSWER

$$\frac{1}{4} = \frac{3t^2}{100} \Rightarrow 100 = 12t^2 \Rightarrow t = \sqrt{\frac{100}{12}} = \sqrt{\frac{25}{3}} = \frac{5\sqrt{3}}{3}$$

6. Consider the function $f(x) = x^2 e^{1/x}$.

[8]   (a) Find the following (no explanation is required in parts i) - iv)):

i. $\lim\limits_{x \to 0^-} x^2 e^{1/x}$

ANSWER:
0

ii. $\lim\limits_{x \to 0^+} x^2 e^{1/x}$

ANSWER:
∞

iii. $\lim\limits_{x \to \infty} x^2 e^{1/x}$

ANSWER:
∞

iv. $\lim\limits_{x \to -\infty} x^2 e^{1/x}$

ANSWER:
∞

v. $f'(x)$

ANSWER:
$f'(x) = (2x - 1)e^{1/x}$

SHOW YOUR WORK

$$f'(x) = 2xe^{1/x} + x^2\left(-\frac{1}{x^2}\right)e^{1/x}$$
$$= (2x - 1)e^{1/x}$$

vi. $f''(x)$

ANSWER:
$f''(x) = e^{1/x}\left[2 - \dfrac{2}{x} + \dfrac{1}{x^2}\right]$

SHOW YOUR WORK

$$f''(x) = 2e^{1/x} + (2x - 1)\left(-\frac{1}{x^2}\right)e^{1/x} = e^{1/x}\left[2 - \frac{2}{x} + \frac{1}{x^2}\right]$$

[5]   (b) Using your findings, find the following.

  i. The x-intercepts and y-intercepts (if any)

  There are none.

  ii. The local maxima and minima (if any)

  $f'(x)=0$, $(2x-1)e^{1/x}=0$ yields a single critical point $x=\dfrac{1}{2}$.

  Since $f''(\dfrac{1}{2}) > 0$, $x=\dfrac{1}{2}$ is a local minimum, $f(\dfrac{1}{2})=\dfrac{\sqrt{e}}{4}$.

  Notice that $f'(0)$ is not defined however $x=0$ is not in the domain of the function.

  iii. The inflection points (if any)

  Consider

$$f''(x) = e^{1/x}\left[2-\frac{2}{x}+\frac{1}{x^2}\right]=0$$
$$\Rightarrow\ e^{1/x}\left[2x^2-2x+1\right]=0$$
$$\Rightarrow\ 2x^2-2x+1 = 0$$
$$\Rightarrow\ x=\frac{2\pm\sqrt{-4}}{4} = 0$$

  This equation has no real roots. Thus, there are no inflection points. Notice that $f''(0)$ is not defined however $x=0$ is not in the domain of the function.
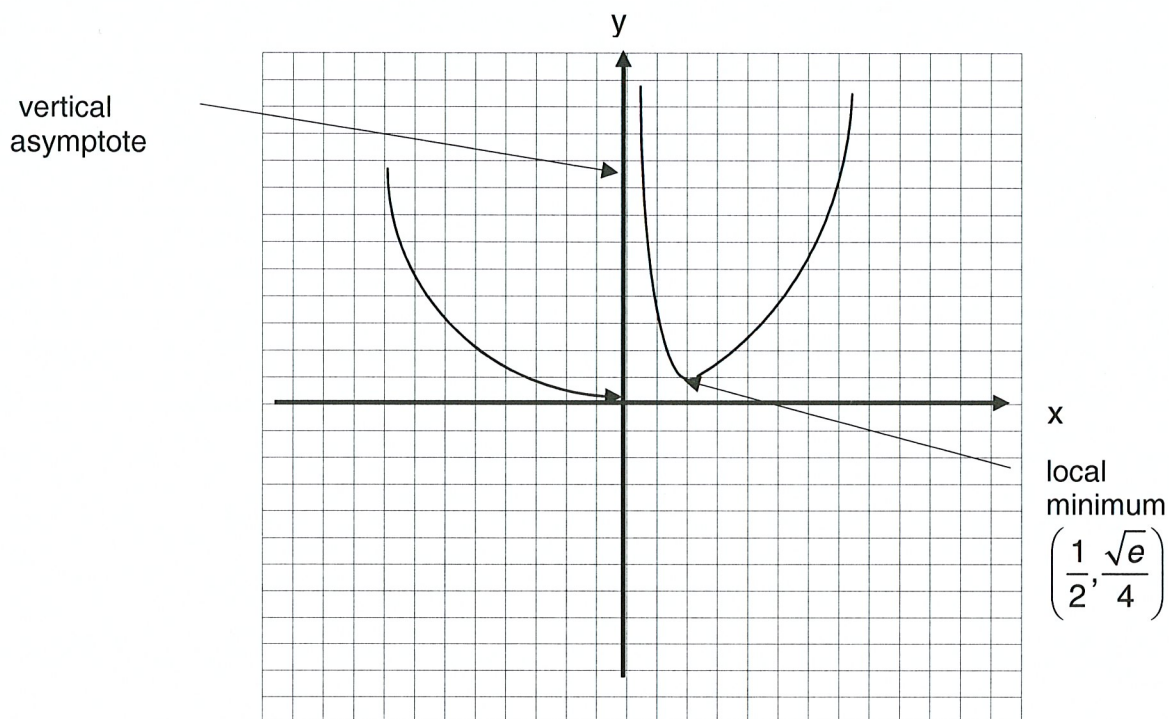
  iv. The largest intervals on which the graph is concave up and concave down

  Since $f''(x)> 0$ for all $x\in(-\infty,0)\cup(0,\infty)$, $f(x)$ is concave up on its domain.

  v. The asymptotes

  It follows from part (a) sections (i) and (ii) that there is a vertical asymptote $x=0$.

[2]   (c) Sketch a rough graph of $f(x)$ indicating your findings:

[4] 7. A copper cube of side 5 *cm* is shaved on all sides to produce a cube of side $(5 - \varepsilon)$ *cm*. Given that the density of copper is 8.96 *g/cm*^{3} and that the shaving decreases the mass of the cube by 0.96 *g*, use a linear approximation to estimate ε.

ANSWER:

$\varepsilon \approx 0.0014$ *cm*

JUSTIFY YOUR ANSWER

Let x be the side length of the cube. Denote density of the copper $\rho = 8.96$ Then the volume of the cube, $V(x)$, and its mass, $m(x)$, are related through the formula: $V(x) = \dfrac{m(x)}{\rho} = \dfrac{m(x)}{8.96}$. Using linear approximation:

$$V(5 - \varepsilon) \approx V(5) + V'(5)[(5 - \varepsilon) - 5]$$
$$V(5 - \varepsilon) \approx V(5) + V'(5)(-\varepsilon)$$
$$V(5 - \varepsilon) - V(5) \approx V'(5)(-\varepsilon)$$
$$\varepsilon \approx \frac{V(5) - V(5 - \varepsilon)}{V'(5)} \approx \frac{\frac{0.96}{8.96}}{V'(5)} \approx \frac{0.107}{V'(5)}$$

Using the fact that $V'(x) = 3x^2$ and correspondingly, $V'(5) = 3(5)^2 = 75$, we get

$$\varepsilon \approx \frac{0.107}{V'(5)} \approx \frac{0.107}{75} \approx 0.0014 \ (cm).$$
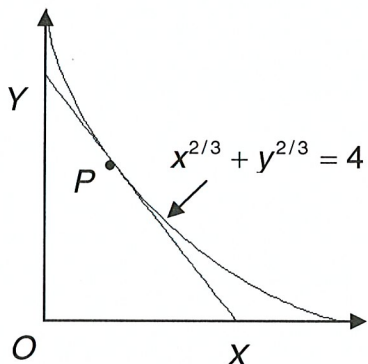
[6] 8. Consider the curve

$$x^{2/3} + y^{2/3} = 4$$

in the first quadrant. Show that the length of a segment XY of a tangent line to the curve at a point P cut off by the coordinate axes is constant and find this length.

> **ANSWER:**
>
> $$XY = 8$$

EXPLANATION

Using implicit differentiation,

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot y' = 0 \;\Rightarrow\; y' = -\left(\frac{y}{x}\right)^{1/3}.$$

Denote the coordinates of the given points: $P(x_0, y_0)$, $X(x_*, 0)$, and $Y(0, y_*)$. Then the slope of the tangent line to the curve at P is found:

$$m = -\left(\frac{y_0}{x_0}\right)^{1/3}.$$

Subsequently, equation of the tangent line to the curve at point P is:

$$y - y_0 = -\left(\frac{y_0}{x_0}\right)^{1/3} (x - x_0).$$

Points $Y(0, y_*)$ and $X(x_*, 0)$ on the graph correspond to $y-$ and $x-$ intercepts of the tangent line, correspondingly. Using the above equation twice gives:

$$y_* - y_0 = -\left(\frac{y_0}{x_0}\right)^{1/3} (0 - x_0), \qquad\qquad 0 - y_0 = -\left(\frac{y_0}{x_0}\right)^{1/3} (x_* - x_0).$$

Simplifying and using the fact that $x_0^{2/3} + y_0^{2/3} = 4$ (point P is on the curve):

$$y_* = y_0 + x_0\left(\frac{y_0}{x_0}\right)^{1/3} = y_0^{1/3}(y_0^{2/3} + x_0^{2/3}) = 4y_0^{1/3},$$

$$x_* = x_0 + y_0\left(\frac{x_0}{y_0}\right)^{1/3} = x_0^{1/3}(y_0^{2/3} + x_0^{2/3}) = 4x_0^{1/3}.$$

Finally, the length of the segment XY of the tangent line

$$XY = \sqrt{x_*^2 + y_*^2} = \sqrt{\left(4x_0^{1/3}\right)^2 + \left(4y_0^{1/3}\right)^2} = \sqrt{16x_0^{2/3} + 16y_0^{2/3}} = \sqrt{16\left(x_0^{2/3} + y_0^{2/3}\right)} = \sqrt{16(4)} = 8.$$

[4] 9. Using logarithmic differentiation, find $y'(0)$ given that

$$y(x) = \frac{\sqrt{1+2x}\,\sqrt[4]{1+4x}}{\sqrt[3]{1+3x}\,\sqrt[5]{1+5x}\,\sqrt[7]{1+7x}}$$

ANSWER:

$$y'(0) = -1$$

SHOW YOUR WORK

$$\ln y = \ln\frac{\sqrt{1+2x}\,\sqrt[4]{1+4x}}{\sqrt[3]{1+3x}\,\sqrt[5]{1+5x}\,\sqrt[7]{1+7x}},$$

$$\ln y = \frac{1}{2}\ln(1+2x)+\frac{1}{4}\ln(1+4x)-\frac{1}{3}\ln(1+3x)-\frac{1}{5}\ln(1+5x)-\frac{1}{7}\ln(1+7x)$$

Differentiate both sides with respect to x:

$$\frac{1}{y}y' = \frac{1}{2}\left(\frac{2}{1+2x}\right)+\frac{1}{4}\left(\frac{4}{1+4x}\right)-\frac{1}{3}\left(\frac{3}{1+3x}\right)-\frac{1}{5}\left(\frac{5}{1+5x}\right)-\frac{1}{7}\left(\frac{7}{1+7x}\right),$$

$$y' = y\left(\frac{1}{1+2x}+\frac{1}{1+4x}-\frac{1}{1+3x}-\frac{1}{1+5x}-\frac{1}{1+7x}\right),$$

$$y'(0) = y(0)(1+1-1-1-1),$$

$$y'(0) = -y(0) = -1.$$

[8] 10. Two carts, *A* and *B*, are on the floor of a warehouse. The carts are connected by a rope 25 metres long. The rope is stretched tight and pulled over a pulley attached to a rafter 3 metres above a point *Q* between the carts (see picture). How fast is the distance *y* between the cart *B* and point *Q* changing when cart *A* is 4 metres away from *Q* and is being pulled away from *Q* at a speed of 2 metres per second?

ANSWER:
  Decreasing at a rate
  $\dfrac{32}{\sqrt{391}}$ metres per second

JUSTIFY YOUR ANSWER
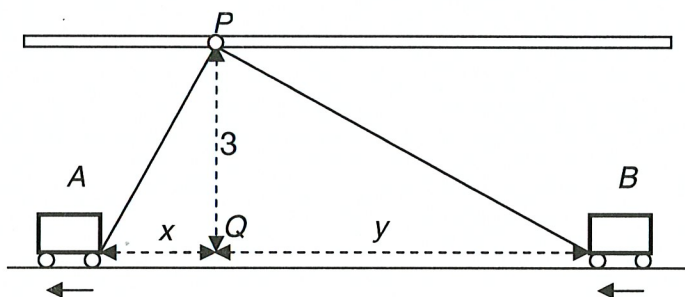
Using the fact that total length of the rope is given to be 25 metres:

$AP + PB = 25,$
$$\sqrt{x^2 + 3^2} + \sqrt{y^2 + 3^2} = 25.$$

Differentiate both sides with respect to time *t*

$$\frac{x}{\sqrt{x^2+9}}\frac{dx}{dt} + \frac{y}{\sqrt{y^2+3^2}}\frac{dy}{dt} = 0. \qquad (**)$$

When $x(t) = 4$, $\dfrac{dx}{dt} = 2$. Immediate consequences of this are:

$AP = \sqrt{16+9} = 5,$
$PB = 25 - 5 = 20,$
$y = \sqrt{PB^2 - 3^2} = \sqrt{20^2 - 9} = \sqrt{391}.$

From (**),

$$\frac{4}{\sqrt{4^2+9}}(2) + \frac{\sqrt{391}}{\sqrt{(\sqrt{391})^2 + 3^2}}\frac{dy}{dt} = 0,$$

$$\Rightarrow \frac{8}{5} + \frac{\sqrt{391}}{20}\frac{dy}{dt} = 0,$$

$$\Rightarrow \frac{dy}{dt} = -\frac{32}{\sqrt{391}}.$$

Thus, the distance *y* between the cart *B* and point *Q* is decreasing at this moment at a rate $\dfrac{32}{\sqrt{391}}$ metres per second.

[5] 11. (a) Find the general solution of the differential equation

$$\frac{dy}{dx} = y(y-2).$$

ANSWER:

$$y = \frac{2}{1 - C_1 e^{2x}}$$

SHOW YOUR WORK

$$\frac{dy}{y(y-2)} = dx$$

$$\Rightarrow \int \frac{dy}{y(y-2)} = \int dx, \quad \frac{1}{2} \int \left[\frac{1}{y-2} - \frac{1}{y} \right] dy = x + C,$$

$$\Rightarrow \ln \frac{y-2}{y} = 2(x+C)$$

$$\Rightarrow \frac{y-2}{y} = e^{2(x+C)}, \quad \frac{y-2}{y} = C_1 e^{2x}, \quad C_1 = e^{2C}$$

$$\Rightarrow y - 2 = C_1 y e^{2x}$$

$$\Rightarrow y(1 - C_1 e^{2x}) = 2$$

$$\Rightarrow y = \frac{2}{1 - C_1 e^{2x}}$$

[2] (b) Find the particular solution of the above equation that satisfies initial condition $y(0) = 0.5$.

ANSWER:

$$y(x) = \frac{2}{1 + 3e^{2x}}$$

SHOW YOUR WORK

$$y(0) = \frac{2}{1 - C_1} = \frac{1}{2}$$

$$\Rightarrow 4 = 1 - C_1$$

$$\Rightarrow C_1 = -3$$

$$\Rightarrow y(x) = \frac{2}{1 + 3e^{2x}}$$

[1]   (c) What happens to the value of $y(x)$ found in Part (b) as $x \to \infty$?

EXPLAIN YOUR ANSWER

$$\lim_{x \to \infty} y(x) = \lim_{x \to \infty} \frac{2}{1 + 3e^{2x}} = 0$$

Hence, the x-axis is a horizontal asymptote on the graph of y(x).

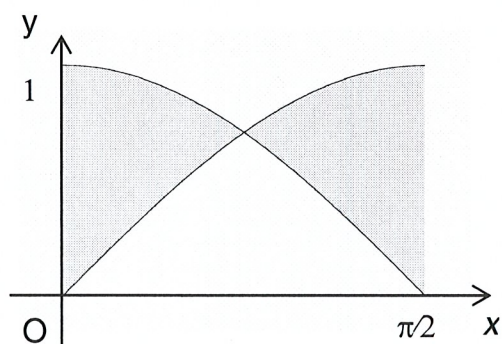[6] 12. Find area of the shaded region included between the curves

$$y = \sin x \ \text{and} \ y = \cos x$$

between $x = 0$ and $x = \pi/2$.

ANSWER:
$$\text{Area} = 2\left[\sqrt{2} - 1\right]$$

SHOW YOUR WORK

$$\text{Area} = 2 \int_0^{\pi/4} (\cos x - \sin x)dx$$

$$= 2\left[\sin x + \cos x\right]_0^{\pi/4}$$

$$= 2\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1\right]$$

$$= 2\left[\sqrt{2} - 1\right]$$

[4] 13. Given that $f''(t) = e^{-2t} + \cos\frac{1}{2}t$ for all t, find an explicit formula for $f(t)$ if $f(0) = f'(0) = 0$.

ANSWER:

$$f(t) = \frac{1}{4}e^{-2t} - 4\cos\frac{1}{2}t + \frac{1}{2}t + \frac{15}{4}.$$

SHOW YOUR WORK

Applying antidifferentiation and satisfying initial conditions twice:

$$f'(t) = -\frac{1}{2}e^{-2t} + 2\sin\frac{1}{2}t + C_1,$$

$$f'(0) = -\frac{1}{2} + C_1 = 0, \qquad\qquad \Rightarrow C_1 = \frac{1}{2},$$

$$f'(t) = -\frac{1}{2}e^{-2t} + 2\sin\frac{1}{2}t + \frac{1}{2},$$

$$f(t) = \frac{1}{4}e^{-2t} - 4\cos\frac{1}{2}t + \frac{1}{2}t + C_2,$$

$$f(0) = \frac{1}{4} - 4 + C_2 = 0, \qquad\qquad \Rightarrow C_2 = \frac{15}{4},$$

$$f(t) = \frac{1}{4}e^{-2t} - 4\cos\frac{1}{2}t + \frac{1}{2}t + \frac{15}{4}.$$

[4] 14. Find

$$\lim_{x \to 0} \frac{\sec x - 1}{x \sec x}.$$

ANSWER:

0

SHOW YOUR WORK

$$\lim_{x \to 0} \frac{\sec x - 1}{x \sec x} = \lim_{x \to 0} \frac{1 - \cos x}{x} = 0.$$

(or using the trigonometric formula for half-angle:

$$\lim_{x \to 0} \frac{\sec x - 1}{x \sec x} = \lim_{x \to 0} \frac{1 - \cos x}{x} = \lim_{x \to 0} \frac{2\sin^2\frac{x}{2}}{x} = \lim_{x \to 0}\left[\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right] \cdot \lim_{x \to 0} \sin\frac{x}{2} = 1 \cdot 0 = 0.$$

[6] 15. Use the chain rule to show that $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$.

EXPLANATIONS

Let $y = \sin^{-1} x$. Then $x = \sin y$.

Differentiate both sides with respect to x.

$1 = \cos y \cdot y' \;\Rightarrow\; y' = \dfrac{1}{\cos y}, \; -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

$\Rightarrow\; y' = \dfrac{1}{\sqrt{1-\sin^2 y}} \quad \text{as } \sin^2 y + \cos^2 y = 1$

$\Rightarrow\; y' = \dfrac{1}{\pm\sqrt{1-x^2}} \quad \text{as } x = \sin y$

However, notice that $\cos y$ is positive when $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$ so we retain only "+" sign.

Thus, $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$, as required.