

This examination has 14 pages including this cover.

# UBC-SFU-UVic-UNBC Calculus Examination

## 9 June 2005, 12:00-15:00

Name: \_\_\_\_\_ Signature: \_\_\_\_\_

School: \_\_\_\_\_ Candidate Number: \_\_\_\_\_

### Rules and Instructions

1. *Show all your work!* Full marks are given only when the answer is correct, and is supported with a written derivation that is orderly, logical, and complete. Part marks are available in every question.
2. Calculators are optional, not required. Correct answers that are “calculator ready,” like  $3 + \ln 7$  or  $e^{\sqrt{2}}$ , are fully acceptable.
3. Any calculator acceptable for the Provincial Examination in Principles of Mathematics 12 may be used.
4. A basic formula sheet has been provided. No other notes, books, or aids are allowed. In particular, *all calculator memories must be empty when the exam begins.*
5. If you need more space to solve a problem on page  $n$ , work on the back of page  $n - 1$ .
6. CAUTION - Candidates guilty of any of the following or similar practices shall be dismissed from the examination immediately and assigned a grade of 0:
  - (a) Using any books, papers or memoranda.
  - (b) Speaking or communicating with other candidates.
  - (c) Exposing written papers to the view of other candidates.
7. Do not write in the grade box shown to the right.

1		5
2		5
3		5
4		5
5		8
6		8
7		8
8		8
9		8
10		8
11		8
12		8
13		8
14		8
Total		100

**UBC-SFU-UVic-UNBC Calculus Examination**  
Formula Sheet for 9 June 2005

**Exact Values of Trigonometric Functions**

$\theta$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	$\pi$
$\sin \theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1/2	$-\sqrt{2}/2$	$-\sqrt{3}/2$	-1

**Trigonometric Definitions and Identities**

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \sin \phi \cos \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

[5] 1. Find the tangent line to the curve  $y = \frac{\sqrt{x}}{1+x^2}$  at the point where  $x = 1$ .

[5] 2. Given  $f(x) = \cos(e^{x^2} + \sin(\ln(x)))$ , find  $f'(x)$ .  
[The correct answer will receive full credit even if no justification is shown.]

[5] **3.** Find the exact value of  $L = \lim_{h \rightarrow 0} \frac{(e^h - 1) \sin(3h)}{h^2}$ .

[5] **4.** Find all points where this curve intersects its horizontal asymptote:

$$y = \frac{x^3 - 2x^2 + 5}{x^2 - x^3}.$$

[8] **5.** Consider this curve  $C: y^4 - 4y^2 = x^4 - 9x^2$ .

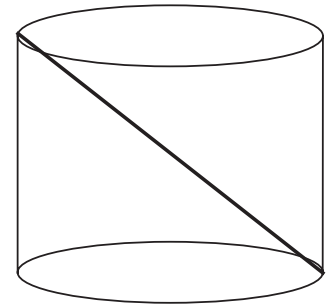
(a) Show that the point  $(3, 2)$  lies on  $C$ .

(b) Use a suitable tangent line to estimate a value of  $b$  for which  $(3.02, b)$  lies on  $C$ .

- [8] **6.** Among all circular cylinders with diagonal length equal to 6, find the height of the one whose volume is largest. Explain how you can be sure that your result gives the *maximum* volume.

*Background:* The diagonal length  $L$  of a cylinder is the length of the longest straight rod that will fit inside it. (See the rough sketch at right.)

*Hint:* Let the desired cylinder have base radius  $r$ , and height  $h$ . Start by finding a relationship between  $r$  and  $h$ , based on the fact that  $L = 6$ .



- [8] 7. A particle travels along the  $x$ -axis. When the particle is at the point  $x$ , its velocity is

$$\frac{dx}{dt} = \frac{1}{1 + 2x}.$$

Find the particle's acceleration at the instant when its position is  $x = 2$ .

- [8] 8. A vegan casserole with a temperature of  $20^\circ\text{C}$  is placed in an oven whose constant temperature is  $200^\circ\text{C}$ . The casserole's temperature reaches  $30^\circ\text{C}$  ten minutes later. How long must the casserole spend in the oven (total cooking time) before it is ready to serve? Base your answer on these assumptions:
- (i) Serving temperature for the casserole is  $90^\circ\text{C}$ .
  - (ii) The rate of change of the casserole's temperature is proportional to the difference between its temperature and the temperature of the oven.

NOTE: A "calculator-ready" answer is fully acceptable.



- [8] **9.** A particle travels in the plane. At each time  $t$ , its  $(x, y)$ -coordinates are given by

$$x = t - \sin t, \quad y = \cos t.$$

How fast is the particle's distance from the origin increasing at the instant when the particle is at the point  $(\pi, -1)$ ?

- [8] **10.** Let  $f(x) = e^{(10x - \frac{x^2}{2})}$ . Find all intervals over which  $f(x)$  is concave up.

- [8] **11.** A particle travels along the  $x$ -axis. At each time  $t$ , the particle's acceleration is

$$a(t) = 4e^{-2t}.$$

At time  $t = 0$ , the particle's velocity is 3, and its  $x$ -coordinate is 7.

- (a) Find a formula for the velocity of the particle at time  $t$ .
- (b) Find the particle's  $x$ -coordinate at time  $t = 10$ .

- [8] **12.** A small company hires students to gather wild mushrooms. If the company uses  $L$  hours of student labour per day, it can harvest  $3L^{2/3}$  kilograms (Kg) of wild mushrooms, which it can sell for \$15.00 per Kg. The company's only costs are for labour. It pays its pickers \$6.00 per hour, so  $L$  hours of labour cost the company  $6L$  dollars.

How many hours  $L$  of labour should the company use per day in order to maximize profit? (Note that daily profit is equal to daily revenue minus daily cost.) Show that your answer really does give maximum profit.

- [8] **13.** There are two lines that are simultaneously tangent to both the curves

$$y = x^2 \quad \text{and} \quad y = -4(x - 1)^2.$$

Find the equations of these two lines.

*Hint:* Make a sketch. Find the equation of the tangent line to the first curve at  $x = a$ , and the equation of the tangent line to the second curve at  $x = b$ .

- [8] 14. The region  $L$  shown in the sketch below lies between the curves  $y = x^p$  and  $y = x^{1/p}$  for some constant  $p > 1$ . It is contained in the square where  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ .

Express the area of  $L$  in terms of  $p$ .

