[12] **1.** In parts (a)–(d) below, algebraic simplification is *not required*:

(a) Calculate $\dfrac{d}{dx}\left(e^x \tan x\right)$.

(b) Find $f'(x)$, given $f(x) = \left(x^2 + \sqrt{\dfrac{x-\pi}{7}}\right)^{2001}$.

(c) Given $g(t) = \sin(2\ln t)$, find $g''(1)$.

(d) Suppose $u = \dfrac{\sin x^2}{1+\cos^2 x}$. Find $\dfrac{du}{dx}$.

[6] **2.** Find all points on the curve $y = \sin^{-1}(x)$ where the tangent line is parallel to the line

$$2x - \sqrt{3}\,y = 100.$$

[6] **3.** Use the definition of the derivative as a limit to find $f'(3)$ for $f(x) = x^{-2}$.
[No marks will be given for an answer obtained using only differentiation rules.]

[4] **4.** Find the positive constant k for which $y = k\sqrt{5x+1}$ satisfies the equation $y\,\dfrac{dy}{dx} = 1$.

[8] **5.** Consider the curve $3^x - 2^y = 1$.

(a) Find the equation of the tangent line at the point $(2, 3)$.

(b) By expressing $\dfrac{dy}{dx}$ as a function of x, or otherwise, find $\displaystyle\lim_{x\to\infty} \frac{dy}{dx}$.

[8] **6.** A ladder 5 metres long is leaning against a high vertical wall when its base begins to slip horizontally away from the wall. The distance s from the base to the wall (measured in metres) satisfies the differential equation

$$\frac{ds}{dt} = 1 + e^{-s}.$$

(Time is measured in seconds.)

Consider the area of the right-angled triangle formed by the ladder, the wall, and the ground. At the instant when the top of the ladder is 3 metres above the ground, ...

(a) is this area increasing or decreasing?

(b) what is the area's exact rate of change? (Give units with your answer.)

[8] **7.** Let $I(x)$ be the amount of light that gets through an x millimeter thick layer of tinted glass. Then $\dfrac{dI}{dx} = -kI$ for some positive constant k.

Continued on page 3

Suppose that a 1 mm layer of the glass allows 60% of the incident light to get through. How thick a layer of the glass should be used so that 1% of the incident light gets through?

[6] **8.** Suppose the function $y = y(t)$ satisfies this differential equation for some $c \geq 0$:

$$y''(t) + cy'(t) + y(t) = 0.$$

Use y to define $E(t) = (y(t))^2 + (y'(t))^2$. Prove that whenever $t_1 < t_2$, we have $E(t_1) \geq E(t_2)$. [Note: It is possible to present a convincing proof without solving the differential equation.]

[6] **9.** A moving particle's displacement s is given by $s(t) = e^{-t} \sin t$ at all times $t > 0$.

(a) For which values of t in the interval $0 < t < \pi$ is the particle's velocity positive?

(b) For which values of t in the interval $0 < t < \pi$ is the particle's acceleration positive?

[8] **10.** A certain function f obeys $f''(x) < 0$ for all x, and $f(2) = 2$. In seeking a zero of f with Newton's Method, the starting point $x_0 = 2$ gives the next guess $x_1 = 1$.

(a) Find $f'(2)$.

(b) With the aid of a suitable sketch, explain why f must have a zero at some point x satisfying $1 < x < 2$.

Your answer should apply to every function f with the properties described above.

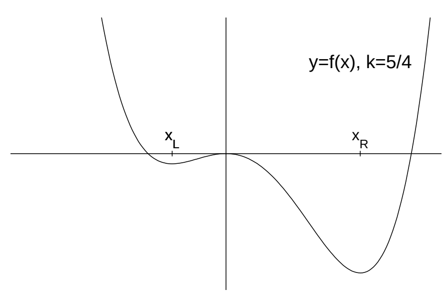
[10] **11.** Find the dimensions of the circular cylinder of greatest volume that can be inscribed in a cone of base radius R and height H, if the base of the cylinder lies in the base of the cone. Express your answer in terms of R and H.

[9] **12.** A certain function f is given. Its second derivative, $f''(x)$, is defined and continuous for all real x. Furthermore, f satisfies $f(-2) = 0$, $f'(-2) = 0$, $f(0) = -2$, $f(4) = 0$. For $y = f(x)$,

$$\begin{aligned} x < -2 &\implies y' < 0,\ y'' > 0, \\ -2 < x < 0 &\implies y' < 0,\ y'' < 0, \\ 0 < x < 2 &\implies y' < 0,\ y'' > 0, \\ 2 < x \quad &\implies y' > 0,\ y'' > 0. \end{aligned}$$

Sketch the curve $y = f(x)$, paying particular attention to slope and concavity. Label any local maxima and minima and points of inflection on your sketch.

[9] **13.** Let $f(x) = 3x^4 + 4(k-2)x^3 - 3kx^2$.
The curve $y = f(x)$ is shown for the case $k = 5/4$.



(a) Find all real numbers k for which f has a local maximum at the point $x = 0$.

(b) Let x_L and x_R denote the x-coordinates of the local minimum points for f, as illustrated in the sketch provided. Assuming $k > 0$, express the separation $s = |x_R - x_L|$ as a function of k.

(c) Among all $k > 0$, which choice minimizes the separation s described in part (b)? Why?

The End

This examination has 3 pages including this cover.

UBC-SFU-UVic-UNBC Calculus Examination

7 June 2001

Name: ______________________ **Signature:** ______________________

School: ______________________ **Candidate Number:** ______________________

Rules and Instructions

1. *Show all your work!* Full marks are given only when the answer is correct, and is supported with a written derivation that is orderly, logical, and complete. Part marks are available in every question.
2. Calculators are optional, not required. Correct answers that are "calculator ready," like $3 + \ln 7$ or $e^{\sqrt{2}}$, are fully acceptable.
3. Any calculator acceptable for the Provincial Examination in Principles of Mathematics 12 may be used.
4. No notes, books, or other aids are allowed. In particular, *all calculator memories must be empty when the exam begins.*
5. If you need more space to solve a problem on page n, work on the back of page $n - 1$.
6. CAUTION - Candidates guilty of any of the following or similar practices shall be dismissed from the examination immediately and assigned a grade of 0:
   (a) Using any books, papers or memoranda.
   (b) Speaking or communicating with other candidates.
   (c) Exposing written papers to the view of other candidates.
7. Do not write in the grade box shown to the right.

1		12
2		6
3		6
4		4
5		8
6		8
7		8
8		6
9		6
10		8
11		10
12		9
13		9
Total		100