Math 100, Section 105
Problem Set 1

Due: September 17th, 2010

Student number:

LAST name:

First name:

Mark:

Instructions

- Please print this sheet out and write your student number, last name (all capitals) and first name. Please use your "official" name as it appears in the student registry even if you prefer to be called by another name — this is needed for the grader to enter your grade in the system.
- Use this page as a cover sheet for your solutions, but do not write your solutions on it. STAPLE your pages together; lost pages are your responsibility.
- Due at the beginning of class on the date indicated; late work will not be accepted.
- Place in the envelope corresponding to the first letter of your last name.

Problems

1. In each of the following cases, decide whether the limit exists. If the limit exists evaluate it, showing your intermediate steps (indicate which limit law you are using at each step). If the limit does not exist, explain why. (a) $\lim_{x\to 1} \frac{x^2-x}{x-1}$; (b) $\lim_{x\to 1} \frac{x^2+x}{x-1}$; (c) $\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{(2+h)^2} - \frac{1}{4}\right)$; (d) $\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$ (here $x > 0$ is fixed) (e) $\lim_{x\to\infty} \frac{x^n+5}{x^m+1}$ for $n, m \geq 1$ fixed (*Hint*: divide through by the smaller of x^n, x^m; your answer will depend on n and m); (f) $\lim_{x\to\infty} x\left(\sqrt{x^2+1} - x\right)$.

(problems 2-3 on the other side) Useful identity for this course: $\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y}\right) = x - y$.

2. According to the theory of relativity a particle moving at the velocity v has mass

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where c is the speed of light and m_0 is a constant depending on the particle, but not on the velocity.

(a) What happens in the limit $v \to 0$? Explain why m_0 is called the *rest mass* of the particle.

(b) What happens when $v \to c^-$?

3. Consider the function $f(x)$ given by

$$f(x) = \begin{cases} ax^3 \cos(\frac{1}{x}) + bx + c & x < 0 \\ Ae^{Bx} + C & x > 0 \end{cases}$$

where a, b, c, A, B, C are constants. Note that f is undefined at $x = 0$.

(a) Find $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$ in terms of a, b, c, A, B, C.

(b) For which values of the constants will there be no "jump" at $x = 0$?

(c) When there is no "jump", what value would you assign $f(0)$ to "fill in the hole"?