Math 100 §105, Fall Term 2010
Midterm Exam

November 8$^{\text{th}}$,2010

Student number:

LAST name:

First name:

Instructions

- Do not turn this page over until instructed. You will have 45 minutes for the exam.
- You may not use books, notes or electronic devices of any kind.
- Solutions should be written clearly, in complete English sentences, showing all your work.
- If you are using a result from the textbook, the lectures or the problem sets, state it properly.

Signature:

1	/18
2	/6
3	/6
4	/10
Total	/40

1 Short-form answers

Show your work and clearly delineate your final answer. Not all problems are of equal difficulty.

[3] a. Differentiate the function $y = \arctan(x)$**; write your answer as a function of** x **alone (you may use the formula** $\frac{d\tan u}{du} = 1 + \tan^2 u$**).**

[3] b. Find the tangent line to $y = x^{\sin x}$ **at the point** $(\pi, 1)$**.**

[3] c. We have $z(t) = e^{x(t)\cdot y(t)}$ **where** x, y **both depend on** t**. If at** $t = 1$ **we have** $x(1) = 0$**,** $x'(1) = 1$**,** $y(1) = 2$**,** $y'(1) = 3$ **find** $\frac{dz}{dt}$ **at** $t = 1$**.**

[3] d. The population of Canada was roughly 18 million in the year 1960, 30 million in the year 2000. Assuming the population grows exponentially, estimate the population of Canada in 2040.

[3] e. Let $f(x) = e^x + e^{-x}$. Use a 2nd order Taylor polynomial to give a rational number approximating $f(\frac{1}{2})$.

[3] f. Show that the error in the approximation is less than $\frac{1}{50}$. You may use the fact that $2 \leq e \leq 3$.

2 Long-form answers

[6] Find the maximum value of $f(x) = x\sqrt{1 - \frac{3}{4}x^2}$ **on the interval** $[0, 1]$**.**

3 Long-form answers

[6] A point is moving on the curve $y^2 = x^3 - 3$ in such a way that the x-co-ordinate is changing at the rate of $3\frac{\textbf{units}}{\textbf{min}}$. How fast is the distance of the point to the origin changing, when the point is at $(2, \sqrt{5})$?

4 Long-form answers

Let $f(x) = e^x - e^{-x}$.

[1] Verify that $\left(f'(x)\right)^2 = 4 + \left(f(x)\right)^2$**.**

[3] Show that f **has an inverse function.**

[3] Let $x = g(y)$ be the inverse function. Find a formula for its derivative in terms of x, y.

[3] Find a formula for $g'(y)$ involving y alone.