Math 105 Week 8 Learning Goals

1 Overview

Over the past few weeks, we have covered numerous techniques for evaluating definite and indefinite integrals in various forms, but these techniques cannot be used on every integral. In these cases we can use numerical integration to approximate the integral and estimate the error in the approximation. The techniques we have covered thus far all involve finite intervals of integration and finite-valued functions. We can, in some cases, evaluate integrals where these conditions are not met, which are called improper integrals. We can apply our understanding of integration to the field of differential equations, which relate an unknown function to its derivatives. They can be used to model many different situations.

Topics to be covered include:

- Absolute vs. relative error (7.7, pp. 557-558)
- Midpoint Rule (7.7, pp 558-559) and Trapezoid Rule (7.7, pp 559-561, e.g. Example 3)
- Calculations involving tables instead of functions (e.g Example 6, p. 563)
- Simpson's Rule (p. 564)
- Errors in the three rules (7.7, p. 565); students can choose between using Δx and $\frac{b-a}{n}$, but they should understand that these are equal. Students do not need to memorize the formulas, they will be provided on exams.
- Improper integrals: infinite intervals (7.8, p. 570-572) (remark: we will not cover solids of revolution) Stress proper notation.
- Improper integrals: infinite discontinuities (7.8, pp. 575-577) Stress proper notation.
- The behaviour of functions of the form x^p, depending on p, near the discontinuity $x = 0$ and over an infinite interval (e.g. Example 2, p. 572)
- Intro to differential equations: what they are, how to verify a solution, solving when one point is known (7.9, pp. 581-583)
- First-order linear differential equations (pp. 584-585)
- Separable first-order differential equations (pp. 585-587)

2 Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives. By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

1. given an approximation c and an exact solution x to a problem, compute the absolute and relative error. [Procedural]

Reading: Text § 7.7 (p. 557)

Practice problems: Text p. 566: 7, 9.

2. give the definition of Midpoint rule , Trapezoidal rule and Simpson's rule to approximate $\int_a^b f(x)dx$.

3. given an even integer n, compute an approximation to $\int_a^b f(x)dx$ using Midpoint rule , Trapezoidal rule and Simpson's rule. [Procedural]

4. calculate and interpret the error bound for an approximation to $\int_a^b f(x)dx$ using Simpson's rule, Midpoint rule, or Trapezoid rule. [Procedural]

You do not need to memorize the formulas given in Theorem 7.2.

5. Recognize and evaluate improper integrals over infinite intervals using the definitions. [Procedural]

The material on solids of revolution will not be covered.

Reading: Text § 7.8 (pp. 570 – 573)

6. recognize and evaluate improper integrals with unbounded integrands using the definitions. [Procedural]

Reading: Text § 7.8 (pp. 595 – 578)

7. describe the concept of a differential equation, including the terms order and general solution. Describe an initial value problem. [Conceptual]

Reading: Text § 7.9 (pp. 581 — 583)

8. find the solution to a first-order linear differential equation and a separable first order differential equation, with or without an initial condition. [Procedural]

Note that differential equations of the form $y'(t) = f(t)$ are separable, as are the first order linear differential equations with constant coefficients covered in the textbook. We will not be covering sketching or interpreting direction fields.

Reading: Text § 7.9 (pp. 584 – 587)

9. model the problem using a differential equation, and use it to solve the problem. [Procedural]