Math 105 Week 7 Learning Goals

1 Overview

Last week we discussed two methods of integration, namely substitution and integration by parts. As we saw, these methods are extremely useful in simplifying and evaluating certain integrals. Unfortunately not all integrals lend themselves to these two strategies. This week, we continue to add a few more techniques to our integration toolkit, which, though not all-inclusive, should vastly increase our repertoire.

Topics to be covered include the following:

- **Review**:
  - $\sin^2 x + \cos^2 x = 1$
  - $\tan^2 x + 1 = \sec^2 x$
  - $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
  - $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- Integrating functions of the form $\sin^a x \cos^b x$, where a and b are any integers (7.3, pp 523-526)
- **Optional:** Reduction formulas for powers of sine and cosine (p. 526). Problems given to students with even powers of sine and cosine will only involve powers that are relatively small, so that reduction formulas are unnecessary.
- Antiderivative of tangent, with proof (7.3, p. 527)
- Antiderivative of secant, proof optional (7.3, p. 528)
- Integrating functions of the form $\tan^a x \sec^b x$, where a is odd and/or b is even (7.3, pp 528-529)
  Remark: we won't cover the reduction formula for even powers of tangent multiplied with odd powers of secant.
- The method of trigonometric substition (7.4, pp 531-536). Students should understand the reasoning behind Table 7.4 (p. 533), but they don't need to worry about the range of theta and they don't need to worry about how we simplify (for example) $\sqrt{\cos^2 \theta} = |\cos \theta| = \cos \theta$ in that range.
- Method of partial fractions for rational functions whose denominators can be factored into (possibly repeating) linear factors. (7.5, pp. 541-543, p. 545)
- **Optional**: method besides substitution for solving system of polynomial equations (7.5, p 544, e.g. Example 2)

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. identify the correct strategy (as outlined in Table 7.2) for computing this integral if a definite or indefinite integral is given and the integrand is a power of sine or cosine functions or product of such powers[Procedural]

Note that the substitution rule (section 5.5), standard trigonometric identities and the half-angle formulae are essential to this section. Make sure that you are familiar with these concepts. It is not necessary to memorize the reduction formulae on page 526. However, the antiderivatives of $\tan x$ and $\sec x$ stated in Theorem 7.1 may be useful later. You should either memorize them or be able to derive them from first principles.

2. find the antiderivative of a product of powers of tangent and secant functions that do not call for reduction formulas, applying the algorithm outlined in Table 7.3. [Procedural]

Note that sometimes, when the integrand is not explicitly of the form stated in objectives 1 and 2, some algebraic manipulation or application of appropriate trigonometric identities reduces it to one of these forms. Recognize forms when such reductions are possible.

3. Recognize functional forms where the computation of the antiderivative demands trigonometric substitution. See Table 7.4. [Procedural]

Reading: Text § 7.4 (pp.531 — 537)

4. Even when the integrand does not contain the giveaway expressions $a^2 \pm x^2$ or $x^2 - a^2$, but involves quadratic functions of x, it may be possible after completing the square followed by subsitution to reduce it to the standard form of Table 7.4. You should be able to recognize integrands that lend themselves to such simplifiction. [Conceptual]

Reading: Text §7.4 (pp. 536—537)

5. Given a rational function $f(x) = \frac{p(x)}{q(x)}$ where q is a polynomial with distinct real roots, be able to describe the main steps of the method of partial fractions involving simple linear factors, and repeated linear factors $(x - r)^m$ with $m = 2$ or 3 as outlined in the procedure table on page 542 and page 545. We will not be covering partial fractions with irreducible quadratic factors (simple or repeated).

6. Occasionally some initial preparation of the integrand is necessary before applying the method of partial fractions. This could involve substitution or long division of the numerator p by the denominator q if the rational function is improper (i.e., degree of p is at least as large as that of q). You should be able to recognize when such a mix of strategies are called for.