Math 105 Week 5 Learning Goals

1 Overview

This week will be about the *Fundamental Theorem of Calculus* that connects differential calculus with integration. We use this result to evaluate definite integrals of functions f that cannot be evaluate with Riemann sums that easily.

We introduce *substitution rule* as a first integration technique to evaluate antiderivatives that are not obvious from our knowledge of differentiation.

Topics to be covered include:

- **Review**: Antidifferentiation (4.8, linked on website)
  Students should *already* know antiderivatives of the following functions: x^n (where n is constant), $\sin x$, $\cos x$, $\sec^2 x$, $\sec x \tan x$, $\csc x \cot x$, $\csc^2 x$, e^x, $\frac{1}{x}$, $\frac{1}{\sqrt{1-x^2}}$, $\frac{1}{1+x^2}$.
- Area function (5.3, p. 363; e.g. Examples 2, 6-7)
- Fundamental Theorem of Calculus, with proof of Part 1 using area functions (pp. 365-366)
- **Optional**: Proof of Part 2 of FTC (pp. 371-372)
- Examples of the use of (both parts of) the FTC (5.3 e.g. Examples 3-5, 7)
- Substitution Rule: motivation by chain rule, Theorem 5.6 (p. 385), procedure (p. 386; e.g. Examples 2-4), use in definite integrals and proper notation regarding bounds (p 388, e.g. Example 5); examples using identities (e.g. Example 6)
- **Optional**: geometric interpretation of substitution (p. 390)

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. give the definition of the area function $A(x)$ for a function $f(t)$ with left endpoint a.

2. compute the area function for a given function $f(t)$ using Riemann sums or geometric interpretation and facts about areas. [Procedual]

Example problem: *Let $f(t) = 2t - 3$. Compute the area function $A(x)$ of $f(t)$ with left endpoint $a = -1$.*

3. give the definition of an antiderivative of a function f.

4. give the definition of an indefinite integral of a function f

5. state the power, constant and sum rule for indefinite integrals. Write down the indefinite integrals of trigonometric functions such as $\sin(ax)$, $\cos(ax)$, $\tan(ax)$, $\sec(ax)$ and apply these to obtain antiderivatives/indefinite integrals for combinations of such functions. [Procedural]

6. explain the arbitrary constant involved in the computation of an indefinite integral. [Conceptual]

7. explain the distinction between a definite and indefinite integral with the same integrand. [Conceptual]

8. recognise that the area function $A(x)$ with left endpoint a of a function $f(t)$ satisfies $A'(x) = f(x)$. [Conceptual]

Reading: Text § 5.3, pp. 363 — 364

9. state the Fundamental Theorem of Calculus. Use the fundamental theorem of calculus to connect a definite integral with the indefinite integral/antiderivative of the integrand. [Conceptual]

Reading: Text § 5.3, Theorem 5.3 Part 1 and 2 (pp. 365 – 366)

10. apply the Fundamental Theorem of Calculus to evaluate definite integrals and compute derivatives of integrals considered as functions in the lower or upper bound. [Procedural]

Reading: Text § 5.3, pp. 367 – 369

Example problem: *Evaluate the integral* $\int_0^{\frac{\pi}{2}} \cos(x)dx$ *with the help of the fundamental theorem of calculus.*

11. state the substitution rule for (in)definite integrals.

12. apply the substitution rule to transform a given (in)definite integral to a more simple (in)definite integral. [Procedural]