Math 105 Week 4 Learning Goals

1 Overview

Over the past few weeks, we have studied vectors and partial derivatives, and how they can be used to optimize functions of two variables. We now return to our attention to functions of one variable, as we start on a new subject, integration, which is a central theme of this course.

This week we will start by trying to measure the area under the graph of a given (non-negative) function. If the graph is just a straight line or a circle, we can use familiar formulas from geometry to find this area. For more complex curves, we will first try to *estimate* the area by splitting it into many small, simple pieces and adding up the areas of each. It will be helpful to have a concise way to express these long sums of many small numbers, so we will learn how to make use of *sigma notation* for this purpose. Finally, we will arrive at the precise area using the concept of *limits* from differential calculus.

This leads to the key concept of the *definite integral* of a function between two points. Although we originally wanted to understand the area underneath a graph, we will see that the definite integral has an intuitive geometric meaning even if the function is negative (and there is no "underneath" to speak of). The definite integral also has applications well beyond the measurement of area, just as the derivative has many uses beyond measuring the slope of a graph.

At first, our ability to actually calculate definite integrals will be rather limited, but we can still study some of the ways that definite integrals behave under various operations like addition, subtraction, and multiplication by constants. Starting next week, we will learn that there is a very close connection between integrals and derivatives, and we will use this relationship to unlock the full power of integration.

Topics to be covered include:

- Approximating areas under curves using Riemann sums: All of Chapter 5.1, except interpretation as displacement (That is, exclude Example 1)

- Definite integrals: all of Chapter 5.2 except general Riemann sums (p. 351)–we will only use Riemann sums with intervals that are all the same length

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. describe a *regular partition* of an interval, a *Riemann sum* for a function on a given interval (including the specific cases of *left*, *right*, and *mid-*

point Riemann sums), and how they can be used to approximate area. [Conceptual]

Example problem: *Sketch an approximation for the area under the graph of $f(x) = x^2$ on the interval $[-2, 2]$ with a regular partition into 4 subintervals, using the midpoint of each subinterval.*

2. compute specific Riemann sums for a function on a given interval. [Procedural]

Example problem: *Calculate the left Riemann sum for $f(x) = \cos(x)$ on the interval $[0, \frac{\pi}{2}]$ using $n = 4$.*

3. read and interpret an expression in sigma notation as the sum of a series of numbers; conversely, be comfortable with using sigma notation to concisely describe a particular sum. [Conceptual]

Example problem: *Express the sum $5 + 7 + 9 + 11 + \cdots + 23 + 25$ using sigma notation.*

4. manipulate expressions written in sigma notation, and to use Theorem 5.1 to evaluate certain sums. [Procedural]

5. express Riemann sums for a function $f(x)$ on a given interval using sigma notation, and identify a function and an interval which give rise to a given Riemann sum in sigma notation. [Conceptual]

6. define a definite integral as the limit of Riemann sums; conversely, be able to recognize a given limit of Riemann sums as corresponding to a definite integral.

Example problem: *Express the integral $\int_1^3 \ln(x)dx$ as a limit of Riemann sums.*

7. interpret a definite integral geometrically as the *net area* (also called *signed area*) of an associated region R, and express the area for a given region as a definite integral. [Conceptual]

8. compute definite integrals of simple functions by using the geometric interpretation and facts about area. [Procedural]

9. use algebraic properties of integrals to deduce the value of one definite integral from information about related integrals. [Conceptual]

10. compute definite integrals of polynomial functions using the limit definition. [Procedural]