Math 105 Week 3 Learning Goals

1 Overview

This week, we will go further into our explorations of maxima/minima of functions of two variables. In particular, we will study absolute maxima/minima (global maxima/minima) and we will learn a procedure to solve constrained optimization problems.

Topics to be covered include:

- Absolute extrema: definition, and finding over closed, bounded regions (12.8, pp 943-947; through example 6)
- **Optional:** Parametrization, e.g 12.8, Example 7, pp. 946-947
- Notation and definition of a gradient
- **Optional:** Geometric interpretation of a gradient (12.6, Theorem 12.11, p. 920)
- **Optional**: "Ballpark Theorem" and proof that the method of Lagrange Multiplers work (12.9, pp 951-953)
- Method of Lagrange Multpliers in Two Variables (12.9, procedure box on p. 953; e.g. Example 1 on pp 953-954)
- Economic models, p. 956

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. give the definition of an absolute maximum/minimum of a function $f(x, y)$. See p. 943.

2. calculate its absolute maximum/minimum values using the procedure outlined on p. 944 in the box if a function $f(x, y)$ on a closed and bounded set in $\mathbf{R}^2$ is given. You are not responsible for finding absolute maxima/minima on open or unbounded sets p. 947. [Procedural]

Example problem: *Find the absolute maximum and minimum values of* $f(x, y) = x^2 + y^2 - 2x + 2y + 5$ *on the set* $R = \{(x, y) : 4 \geq x^2 + y^2\}$.

3. describe constrained optimization with the terms *objective function* and *constraint*.

4. describe the notation $\nabla f =< f_x, f_y >$ used for the gradient vector of a function, and compute it for a given function $f(x, y)$ in two variables. [Familiarity with Notation and Terminology]

5. use the method of Lagrange Multipliers given in the box on p. 953 to solve the constrained problem in two variables. You will not need to consider more than 2 variables (i.e. ignore Lagrange Multipliers with Three Independent Variables beginning on p. 955). [Procedural]

Example problem: *Find the maximum and minimum values of the objective function* $f(x, y) = 2x^2 + y^2 + 2$, *where* x *and* y *lie on the ellipse* $\mathcal{C}$ *given by* $g(x, y) = x^2 + 4y^2 - 4 = 0$.

Note that you are also responsible for the section on Economic Models on p. 956.