

Math 105 Week 2 Learning Goals

1 Overview

This week, we will be learning about *partial derivatives* of functions of two variables. These are analogs of the derivative of a function of a single variable, and many aspects of these will be familiar. However, there are certain subtleties which must be addressed.

We'll then begin to look at applications of partial derivatives to the study of certain real-world problems.

Finally we will apply partial derivatives to study maxima and minima of functions of two variables. Again, there are similarities to the use of derivatives in studying extrema of functions of a single variable, but we will see that surfaces exhibit a greater diversity of behaviours than simple curves.

Topics to be covered include:

- Definition of a partial derivative (12.4, pp 895-897)
- Computing partial derivatives by viewing certain variables as constant (12.4, p 898)
All techniques from first-semester calculus can be applied. This includes the power, product, quotient, and chain rules; derivatives of trigonometric, inverse trigonometric, exponential, and logarithmic functions; logarithmic differentiation; and implicit differentiation.
- Higher-order partial derivatives: notation, computation (12.4, pp 899-900, Table 12.4 and Example 4)
- Clairaut's Theorem (students do not need to be able to determine whether mixed partials are continuous; enough to explain to them that it works when the derivatives are well-behaved) (12.4, p 900)
- Simple word problems involving interpretations of partial derivatives (12.4, p 901)
- Definitions of local extrema and Theorem 12.13 (12.8, p 939)
- Critical points and saddle points (12.8, p 940)
- Second Derivative Test (12.8, pp 941-942)
- Optimization (12.8, e.g. Example 4, pp 942-943)

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. give the limit definition of the partial derivatives f_x and f_y of a function $f(x, y)$ of two variables at a point (a, b) .
Example problem: If $f(x, y) = 2x^2 - 3y^2 - 2$, write the limit definition of the partial derivative f_x .
2. describe the meanings of, and give interpretation of, the partial derivatives f_x and f_y in terms of the meanings of $f(x, y)$, x , and y , where $f(x, y)$ is a function representing some physical or mathematical relationship. [Conceptual]
Example problem: The production P of a given factory is described as a function of capital investment K (measured in dollars) and labour L (measured in worker hours.) Give an economic interpretation of the partial derivatives $\frac{\partial P}{\partial K}$ and $\frac{\partial P}{\partial L}$.
3. compute the first and higher partial derivatives of a function of two variables. [Procedural]
Example problem: Let $f(x, y) = x \sin y$. Find f_x and f_y . Then find all of the second partial derivatives of f .
4. state and apply Clairaut's theorem. [Conceptual]
Example problem: Given two functions $f(x, y) = 2y$ and $g(x, y) = 3x$, determine whether there exists a function $F(x, y)$ such that $F_x = f$ and $F_y = g$ (Clairaut's theorem) [Conceptual]
5. give the definition of a local minimum (respectively, maximum) of a function $f(x, y)$ of two variables.
6. state a theorem which gives criteria for a function $f(x, y)$ to have a local minimum (maximum) at a point (a, b) , in terms of the value of the partial derivatives f_x and f_y at (a, b) . [Conceptual]
7. explain how to use the theorem in the previous learning objective to find potential local maxima and minima for a differentiable function $f(x, y)$. [Conceptual]
8. explain what is required for a point (a, b) to be a critical point of a function $f(x, y)$.
9. find critical points of a given function $f(x, y)$ of two variables. [Procedural]
10. give the definition of a saddle point for a function $f(x, y)$, and explain the significance of such points. [Conceptual]
11. classify critical points of a given function $f(x, y)$ using the second derivative test. [Procedural]
12. solve word problems involving the finding of maximum or minimum values. [Procedural; Problem Solving]

Note that The theory and techniques involved in finding absolute maxima and minima (pp. 943 – 948) will be discussed in Week 3; skip this section of the reading and problems for now.