

Math 105 Week 1 Learning Goals

1 Overview

In this introductory week, we will define scalar functions of several variables, and discuss their geometric interpretations. This is a subset of the material contained in sections 12.1 and 12.2 of the textbook. A rudimentary knowledge of vectors and vector arithmetic will be useful for this discussion, and will be presented briefly. Topics to be covered include:

- Vectors: difference between a vector and a point; drawing vectors in \mathbb{R}^3
- Vector addition and subtraction, scalar multiplication (11.1, p.762; 11.2 p 775)
- finding the length of a vector; the direction of a vector (unit vector in same direction) (11.2 p 776-777, parts b and c of Example 6);
- calculating the distance between two points in three-dimensional space (11.2, p 773: this will be useful for understanding the equation of a plane)
- Dot product: determining whether two vectors are orthogonal (using the dot product), parallel, or neither (11.3, pp.782-783)
- Equation of a plane passing through a point and perpendicular to a vector (12.1, pp. 858-859)
- Definition of normal vector to a plane (12.1, pp. 858-859)
- Orthogonal and parallel planes (12.1, p861)
- Trace of a surface (12.1, p862)
- Functions, domain and range with two independent variables (explain independent vs. dependent variables) (12.2, pp. 873-874)
- Graphs of functions of two variables (12.2, p874)
- Level curves (12.2, pp. 876-880)
Students should be familiar with the following curves: points, lines, parabolas, circles, and ellipses.

The following topics are not in the syllabus: finding the angle between two vectors (unless that angle is right or straight), equation of a plane passing through three points (in particular, cross product of two vectors), quadric surfaces, identification of surfaces (such as cylinder, ellipsoid, paraboloid, hyperboloid) from their defining equations, functions of more than two variables.

While you will not be asked to identify a surface in a test, you should use the content of these lectures to develop the skill of sketching at least roughly the shape of a surface given certain standard equations, such as

$$x^2 + y^2 + z^2 = 1, \quad z = \frac{x^2}{a^2} + \frac{y^2}{b^2}, \quad z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$$

This will be a very useful tool in later sections.

2 Learning objectives

By the end of the week and after going through the practice problems, you should be able to:

1. distinguish (notationally and conceptually) points and vectors in three-dimensional space,
2. add and subtract vectors,
3. multiply a vector by a number (scalar),
4. identify if two vectors are parallel, orthogonal or neither,
5. find the length and direction of a vector,
6. find the distance between two points in three-dimensional space,
7. identify a linear equation in three variables as a plane in three-dimensional space,
8. read off a normal vector to a plane from its defining equation,
9. decide whether a point lies in the plane or not,
10. check if two planes are parallel, orthogonal or neither,
11. given an equation in variables x , y , z , identify and sketch the traces of the surface it represents,
12. use the traces to obtain a rough shape of the surface,
13. find the domain and range of a function of two variables,
14. know the definition and geometric significance of level curves,
15. draw the level curves of a function of two variables, when those curves are one of the following types of equations: points, lines, parabolas, ellipses.