Math 105 Week 11 Learning Goals

1 Overview

Last week we started on infinite series. In particular we learned about two tests of convergence and/or divergence: namely the divergence test and the integral test. This week we will learn a few more such tests designed to verify convergence or divergence of an inifinite series, even when we cannot evaluate the series exactly. We will then begin the study of a very special class of functions whose functional forms are given by infinite series.

- Ratio test, comparison test, and limit comparison test (8.5, pp. 641-647) (If students haven't already been introduced to factorials, now is the time.) Remark: we won't cover the root test
- Absolute and conditional convergence: definition, and theorem that absolute convergence implies convergence (Theorem 8.21, p. 654).
- Using Theorem 8.21 to apply tests that only work with positive terms to series with mixed-sign terms. (e.g. Example 3(c), p. 655) Remark: we will not cover alternating series.
- Informal definition of a power series and the centre of a power series (9.1, p. 661)
- **Review**: Taylor Polynomials (9.1, pp. 664-668)
- Taylor's Remainder Theorem (9.1, p. 668) and estimating the remainder (9.1, p. 669)
- Dfinition of a powers series and associated coefficients, centre, interval of convergence, and radius of convergence (9.2, p. 676)
- Theorem 9.3, Convergence of Power Series (9.2, p. 678); we won't concern ourselves with the endpoints of the interval of convergence, but students should be able to find the maximum open interval over which the power series converges.
- Combining power series, Theorem 9.4 (9.2, p. 679)
- Differentiation and integration of power series, Theorem 9.5 (9.2, p. 680)

2 Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives. By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

1. state the ratio test, comparison test and limit comparison test for convergence or divergence of an infinite series with positive terms. [Recall/Conceptual]

2. use the tests learned so far to check if an infinite series converges or diverges. Be able to recognize forms that indicate the applicability or otherwise of certain tests. [Procedural]

3. give the definition of absolute and conditional convergence. Give examples of some absolutely convergent and some conditionally convergent series (p.654). [Recall] Reading: Text §8.5 (pp. 641 – 647, excluding the portion on root test)

4. given a function f that can be differentiated up to any order, describe the nth-order Taylor polynomial for f with center a. [Procedural]

5. given a function f that can be differentiated up to any order, explain why the nth order Taylor polynomial may be considered a good approximation to f. Use Taylors theorem and Taylors remainder formula to estimate the error in approximation of f. [Procedural/Conceptual]

6. define a power series, and the radius of convergence of a power series. Use the radius of convergence to determine an interval where the series converges absolutely. [Recall]

7. given a power series, be able to find the radius of convergence of a power series. [Procedural]

8. Be comfortable with algebraic manipulations of power series, such as sums, differences, multiplication by a power, composition, integration and differentiation. [Procedural]

Reading: Text § 9.2 (pp. 675 – 682)