Math 105 Week 10 Learning Goals

1 Overview

In first-semester calculus, you learned what it meant to talk about the limit of a function. We begin this week by discussing what it means to talk about the limit of an infinite list of numbers (which we call an infinite sequence). For example, the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ converges to zero while both of the sequences $0, 1, 2, 3, \ldots$ and $-1, 1, -1, 1, \ldots$ fail to tend to a limit (in other words, diverge).

The next question we turn to is: Given an infinite list of numbers, what does it mean to add them all up? (A sum of an infinite list of numbers is called an infinite series.) We discuss certain special types of infinite series (such as geometric series, telescoping series, and p-series), as well as tests that allow you to determine convergence and divergence for very general series.

Topics to be covered include:

- Examples and definition of a sequence (8.1, pp. 596-597)
- Difference between an explicit formula and a recurrence relation (p. 597–*finding* explicit formluas from recurrence relations is not emphasized)
- Practice working with sequences (e.g. Examples 1-3, pp. 597-598)
- Limit of a sequence (p. 599 - informal definition only, no epsilons; e.g. Examples 4-6, pp. 599-600)
- Infinite series and partial sums (nice examples: geometric example p. 601; Example 7 p. 601; definitions: p. 602)
- Limit of a sequence of partial sums (e.g. Example 8, p. 603)
- Finding limits of sequences: treating them as functions; using limit laws (8.2, p. 607)
- Terminology for sequences (8.2, p. 608)
- Geometric sequences and their convergence (8.2, pp. 609-610)
- Squeeze Theorem for sequences (8.2, p. 611)
- Bounded Monotonic Sequence Theorem (8.2, p. 612)
- **Optional**: Growth rates of sequences, pp. 613-614
- Finite geometric sums; Geometric series and their convergence and manipulation in sigma notation (8.3, pp. 619-622)
- Telescoping series (8.3, p. 622)

- Divergence test (and example of Harmonic Series that its converse it not true) (8.4, pp 627-629)
- Integral Test, and application to p-series (pp. 630-633)
- **Optional**: Estimating a series with positive terms
- Properties of convergent series (p. 636)

2 Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of objectives. By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to independently achieve all of the objectives listed below.

1. give the definition of a sequence. [Conceptual]

2. compute the first several terms of a sequence given to you either by an explicit formula or by a recursive definition. [Procedural]

3. explain what it means for a sequence to have a limit (converge) or diverge. Given several terms of a sequence, be able to formulate a guess as to its limit. [Conceptual + Procedural]

You do not have to understand the formal definition of the limit of a sequence presented on p. 615.

4. [Procedural] know how to compute limits of sequences using either of the following methods:

- Writing the nth term as $f(n)$, where f is a function for which $\lim_{x\to\infty} f(x)$ exists. See Theorem 8.1 on p. 607.
- Applying the squeeze theorem. See § 8.2 (p. 611).

5. apply the properties of limits summarized in Theorem 8.2 on p. 607. [Procedural]

6. recognize when a sequence is increasing, decreasing, bounded, or monotone. [Conceptual]

7. be able to state Theorem 8.5: A bounded monotonic sequence converges. Recognize (using the skills acquired in Objective 2) when this result applies. [Conceptual]

8. recognize examples of geometric sequences and determine whether they converge or diverge. [Conceptual and Procedural]

9. give the definition of an infinite series $\sum_{k=1}^{\infty} a_k$ and explain what is meant by the **sequence of partial sums**. Relate the convergence or divergence of the series to the sequence of partial sums. [Conceptual]

10. recognize when a geometric series converges and be able to compute its sum in that case. [Conceptual + Procedural]

11. be able to manipulate convergent series according to the rules described in Theorem 8.13 on p. 636. [Procedural]

12. [Procedural] be able to use both of the following tests:

- The divergence test (Theorem 8.8, p. 627), which gives a sufficient condition for a series to diverge.
- The integral test (Theorem 8.10, p. 630), which shows the equivalence between the convergence of a series and that of an associated integral.