Elasticity of Demand Problems
MATH 104 and Math 184
November 8, 2011

1. The current toll for the use of a highway is \$2.50. Drivers use this highway because of its convenience even though there are other routes that are free. The provincial government does a study that determines that a toll of p dollars means q cars will use the road, where

$$q = 60000e^{-0.5p}.$$

Compute the elasticity ϵ at $p = 2.50$ and use it to determine whether an increase in the toll will increase or decrease revenue.

ANSWER: $\epsilon(p) = -\frac{p}{2}$, so $\epsilon(2.50) = -1.25$. Since $|\epsilon(p)| > 1$, decreasing price increases revenue.

2. Currently 1800 people ride a commuter passenger ferry each day and pay \$4 for a ticket. The number of people q willing to ride the ferry at price p is determine by the relationship

$$p = \left(\frac{q - 3000}{600}\right)^2.$$

The company would like to increase its revenue. Use the price elasticity of demand ϵ to give advice to management on whether it should increase or decrease its price per passenger.

ANSWER: $\epsilon(4) = -\frac{1}{3}$. This means raising the price will increase revenue.

3. A certain commodity satisfies the demand equation relating price, p, and quantity demanded, q,

$$q = \frac{1000}{p^2}.$$

If the price of this commodity is lowered, will the revenue generated by its sales increase?

ANSWER: $\epsilon(p) = -2$, which is constant for all p. Since $|-2| > 1$, decreasing the price would mean an increase in revenue.

4. The price p (in dollars) and the demand q for a product are related by

$$p^2 + 2q^2 = 1100.$$

If the current price per unit is \$30, will revenue increase or decrease if the price is raised slightly?

ANSWER: Note that $p = 30$ corresponds to $q = 10$. Use implicit differentiation to get dq/dp. $\epsilon(30) = -\frac{9}{2}$. The demand for this product is elastic, so increasing the price slightly will decrease revenue.

5. A cell phone supplier has determined that demand for its newest cell phone model is given by

$$qp + 30p + 50q = 8500,$$

where q is the number of cell phones the supplier can sell at a price of p dollars per phone. If the current price is \$150, will revenue increase or decrease if the price is lowered slightly? What price should the cell phone supplier set for this cell phone to maximize its revenue from sales of the phone? Use the price elasticity of demand to solve this problem.

ANSWER: Note that $p = \$150$ corresponds to $q = 20$. Use implicit differentiation to find dq/dp. $\epsilon(150) = -\frac{15}{8}$. Demand is elastic, so a slight price decrease will increase revenue. Solve $\epsilon(p) = -1$ to find that price $p = \$80$ maximizes revenue.

Continuous Compound Interest Problems

MATH 104 and Math 184

October 15, 2012

1. Find the present value of \$5000 to be received in 2 years if the money can be invested at 12% annual interest rate compounded continuously.

2. An investment earns at an annual interest rate of 4% compounded continuously. How fast is the investment growing when its value is \$10 000?

3. One thousand dollars is deposited in a savings account at 6% annual interest rate compounded continuously. How many years are required for the balance in the account to reach \$2500?

4. In a certain neighbourhood of Vancouver, property values tripled from 2001 to 2011. If this trend continues, when will property values be five times their 2001 level? Assume property values behave as if the annual investment rate is compounded continuously.

5. Suppose that the present value of \$1000 to be received in 5 years is \$550. What rate of interest, compounded continuously, was used to compute this present value?

6. Investment A is worth \$70 thousand, and is growing at a rate of 13% per year compounded continuously. Investment B is worth \$60 thousand is growing at a rate of 14% per year compounded continuously. After how many years will the two investments have the same value?

ANSWERS:

1. $\$5000e^{-.12(2)}$
2. $\$10000(.04)$
3. $\frac{\ln(2.5)}{.06}$ yrs
4. $\frac{10\ln(5)}{\ln(3)}$ yrs after 2001
5. $\frac{\ln(\frac{100}{55})}{5} 100$ %
6. $100\ln(\frac{7}{6})$ yrs