MATH 184: Week 0 Learning Goals

In this introductory week, we will review some material on exponential functions, logarithms, and inverse functions. This material is found in Chapter 0, pp141 to 143 and Appendix A. The natural exponential and logarithm functions are extremely important in this course. Note that we will revisit these functions in more detail later in the course, but it is good to have early exposure and multiple hits at this material.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain what an exponential function is and what a logarithmic function is. You should know the basic properties of exponential functions and logarithmic functions on pp141 to 143.

2. explain what a one-to-one function is and how to test for this property graphically using the Horizontal Line Test.

3. explain what an inverse function is. Given the graph of a function, you should be able to graph the inverse function, if it exists.

MATH 184: Week 1 Learning Goals

This week we introduce some concepts from business: revenue, costs, and profit, and introduce the students to their first optimization problem. This problem motivates the need for the derivative. Before formally introducing the derivative (week 2), we cover the material of Chapter 1.1 to 1.4. We have skipped Definition 1.3.10 (infinite limits) and Chapter 1.5 (limits at infinity) and will return to these section when we do curve sketching in Week 9. Please note that Theorem 1.4. 17 (Squeeze Theorem) is not covered in Math 184. The specific learning goals for this week are that by the end of the week, you should be able to:

1. explain revenue, costs, and profit for the case of linear demand. You should be able to set up and solve a simple problem involving maximizing revenue, for example, in this context. Note there are some posted notes on this material on the common Math 184 website.

2. compute the limits of functions, as presented in Chapter 1.4.

3. explain one-sided limits and their relationship to two-sided limits. You should be able to examine these limits graphically and numerically.

MATH 184: Week 2 Learning Goals

This week we introduce continuity and the derivative. This is the material in Chapter 1.6, Chapter 2.1 to 2.3 of. The specific learning goals for this week are that by the end of the week, you should be able to:

1. explain what it means for a function to be continuous at a point. you should be able to correctly analyze whether a given function is continuous at a given point.

2. identify points of discontinuity for a given function.

3. know the way continuous functions behave under basic algebraic operations, and use these results to correctly identify whether or not a given function is continuous at a point.

4. know the way continuous functions behave when they are composed.

5. identify whether or not a given function is continuous on a given interval. This includes identifying when a function is left- or right-continuous at the endpoints of a closed interval.

6. state the Intermediate Value Theorem and to apply it to simple situations such as determining whether or not a function has a zero in some interval.

7. explain the notion of the slope of the tangent line at that point and find the equation of the tangent line to a given function at a given point (Theorem 2.3.2).

8. state the definition of the derivative and use it to compute the derivative of a given function in simple cases (Chapter 2.2).

MATH 184: Week 3 Learning Goals

This week we introduce some of the basic rules of differentiation. This is the material in Chapter 2.4, 2.6 , and 2.7.

The specific learning goals for this week are that by the end of the week, you should be able to:

1. use the power, sum, and constant multiple rules to differentiate, for example, polynomials.

2. correctly state and use the product rule.

3. correctly state and use the quotient rule.

4. use the derivative of an exponential function and know the definition of "e" as the base of the exponential function with the property that $\frac{d}{dx}(e^x) = e^x$ (Theorem 2.7.4).

5. differentiate given functions using appropriate combinations of the rules of differentiation.

MATH 184: Week 4 Learning Goals

This week we will introduce derivatives of the basic trigonometric functions, but only as a table of derivatives (Theorem 2.8.5). We work with the chain rule in Chapter 2.9.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. compute derivatives involving the basic trigonometric functions.

2. state the Chain Rule, including its hypotheses, and identify when it can be used.

3. make use of the Chain Rule in computations.

MATH 184: Week 5 Learning Goals

We will cover the natural logarithm (Chapter 2.10), implicit differentiation (Chapter 2.11) and higher order derivatives (Chapter 2.14) this week.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. use the derivative of the natural logarithmic function in computations (Theorem 2.10.1).

2. use the derivatives of general exponential functions and general logarithmic functions in computations (Corollary 2.10.5).

3. use the the technique of logarithmic differentiation to find the derivatives of product (or ratio) functions.

4. explain what we mean by implicit differentiation and identify situations where they will use it.

5. carry out computations involving implicit differentiation.

6. find equations of tangent lines to graphs of implicitly defined functions.

7. compute higher-order derivatives (Chapter 2.14).

MATH 184: Week 6 Learning Goals

This week we will cover *price elasticity of demand*, which is not covered in the online textbook. While such elasticities can be a big topic, we will cover it in an introductory way. You find the material in the notes we have post online on price elasticity of demand. Finally, we will introduce exponential growth and decay, especially as applied to continuous compound interest. This material are in Chapter 3.3.1, Chapter 3.3.3 and **Supplementary notes**. which also give an extra set of problems for the price elasticity of demand and for the continuous compound interest.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. compute the *price elasticity of demand* and use it to determine the direction revenue changes when there is a change in price.

2. solve problems involving price elasticity of demand.

3. solve problems involving exponential growth and decay (Chapter 3.3.1 and Chapter 3.3.3).

4. solve problems involving continuously compounded interest.

MATH 184: Week 7 Learning Goals

This week we cover two topics. The first topic is Related Rates, which is in Chapter 3.2 . The second is Maxima and Minima, which are covered in Chapter 3.5.1 and 3.5.2.

The specific learning goals for this week are that by the end of the week, you should be able to:

1. set up and solve related rates problems; [3.11: 3, 10, 15, 19, 22, 24, 29.46]. By this, we mean: Given a related rates problem, or provided a model (including equation) of another situation, you should be able to:

(a) identify all the variables involved, make appropriate choices when a variable takes on constant values, and describe how they relate (using equations if relevant and/or writing a short paragraph);

(b) draw a picture of the situation if needed;

(c) interpret rates in terms of derivatives with the appropriate variables; and

(d) derive an equation which describes how the relevant rates are related and solve in that equation for the desired target rate.

2. define local maximum and local minimum and give examples of functions that illustrate these.

3. define absolute maximum and absolute minimum and give examples of functions that illustrate these concepts.

4. define a critical point, a singular point and apply these definitions to find critical points and singular points of a given function.

5. find the absolute maximum and absolute minimum of a given continuous function on a closed interval.

MATH 184: Week 8 Learning Goals

This week we cover the first and second derivative tests and curve sketching. This is material in Chapter 3.6, with some additional material from Definition 1.3.10 (infinite limits) and Chapter 1.5 (limits at infinity) focused on asymptotes.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. understand infinite limits (Definition 1.3.10) and limits at infinity (Chapter 1.5).

2. identify any asymptotic behaviours a function may have: vertical asymptotes and horizontal asymptotes (Chapter 3.6.1).

3. explain how the first derivative of a function determines where the function is increasing and decreasing and apply this to specific functions to determine their intervals of increase and decrease (Chapter 3.6.2).

3. explain how the second derivative of a function determines concavity and apply this to specific functions to determine where they are concave up and concave down (Chapter 3.6.3).

MATH 184: Week 9 Learning Goals

We will finish our treatment on Graphing Functions (Chapter 3.6.4 to 3.6.6).

The specific learning goal for this week is that by the end of Week 9, students should be able to use calculus to sketch a graph of a given function.

MATH 184: Week 10 Learning Goals

We will start to work with Optimization Problems in Chapter 3.5.3 in Week 10. There will be some extra business related optimization problems posted as well.

The specific learning goals for this week are that by the end of Week 8, students should be able to:

1. interpret the idea of optimization as the procedure used to make a system or a design as effective or functional as possible, and translate it into a mathematical procedure for finding the maximum/minimum of a function.

2. set up an optimization problem by identifying the objective function and all appropriate constraints.

3. use calculus to solve optimization problems.

MATH 184: Week 11 and 12 Learning Goals

Over the last two weeks of term, we will cover Approximation (linear, quadratic and the more general Taylor polynomials), and finish with a discussion on inverse trigonometric functions. The linear approximation, the quadratic approximation, the Taylor polynomial material and the error in Taylor polynomial Approximation are in Chapter 3.4. The inverse trigonometric functions are in Chapter 2.12. You should have covered the basics of inverse trigonometric functions in Math 12, and the main content of Chapter 2.12 for you will be the derivatives of $arcsin\, x$, $arccos\, x$, and $arctan\, x$.

The specific learning goals for this week are that by the end of week 12, you should be able to

1. explain linear approximation (also known as tangent line approximation and the linearization of a function) , and understand what is the role of a, what is the role of x, and where is the appropriate tangent line on the graph.

2. use linear approximations to estimate the values of functions near a given $x = a$.

3. use linear approximation to approximate changes in the dependent variable given changes in the independent variable (Chapter 3.4.6).

4. given the exact value, discuss the discrepancy with the linear approximation in terms of the second derivative (for example, whether it is an underestimate or overestimate) (Chapter 3.4.8).

5. use the quadratic approximation to estimate the values of functions.

6. find the nth degree Taylor polynomial of a given function with a given centre $x = a$.

7. use the table of derivatives of inverse trigonometric functions in calculations of derivatives (Theorem 2.12.7).