The University of British Columbia

October 16, 2018

Common Midterm 1 for Sections of MATH 184 (Version 1)

Closed book examination — Time: 60 minutes

Last Name ______________________ **First** ______________

Signature ______________

Student Number ______________

MATH 184 **Section Number:** ________

Special Instructions:

No memory aids are allowed. No calculators. No communication devices. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		15
2		5
3		8
4		6
5		11
6		5
Total		50

Short-Answer Questions: Put your answer in the box provided. Full marks will be given for a correct answer placed in the box, while part marks may be given for work shown. Unless otherwise stated, calculator ready answers are acceptable.

[15] **1.**

(a) [3] The height in feet of a baseball hit straight up from the ground with an initial velocity of 16 ft/s is given by $h = f(t) = 16t - 4t^2$, where t is measured in seconds after the hit. Is this function $h = f(t) = 16t - 4t^2$ one-to-one on the interval $0 \leq t \leq 4$?

Answer:

(b) [3] If

$$\lim_{x \to a^-} f(x) = L \qquad \text{and} \qquad 3 \lim_{x \to a^+} f(x) = M,$$

where L and M are finite real numbers, then how are L and M related if $\lim_{x \to a} f(x)$ exists?

Answer:

(c) [3] Suppose

$$f(x) = \begin{cases} 3 & \text{if } \ x \leq 3 \\ x+5 & \text{if } \ x > 3 \end{cases}.$$

Let $A = \lim\limits_{x \to 3^-} f(x)$, $B = \lim\limits_{x \to 3^+} f(x)$ and $C = \lim\limits_{x \to 0} f(x)$. Find A, B and C.

Answer:
$A =$, $B =$, $C =$

(d) [3] Suppose $f(2) = -1$ and $f'(2) = 3$. Let $g(x) = x^2 f(x)$. Find $g'(2)$.

Answer:

(e) [3] Evaluate $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$.

Answer:

Full-Solution Problems: Justify your answers and show your work. Place a box around your final answer. Unless otherwise indicated, simplification of answers is required in these questions.

[5] **2**. Let $f(x) = \dfrac{3}{5x+2}$. Use the definition of the derivative to find $f'(-1)$. No marks will be given for the use of any differentiation rules.

[8] **3.**

(a) Explain why the following equation

$$\sin\left(\frac{\pi}{4}x\right) - x^2 + \frac{1}{2} = 0.$$

has a solution. You can use the fact: $\sin(ax)$ and $\cos(ax)$ are continuous on its domain, where a is a constant.

(b) Determine the value of the constant a which makes the function

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1} & \text{if } x \neq -1 \\ a & \text{if } x = -1 \end{cases}$$

is continuous at $x = -1$.

[6] **4.**

(a) Suppose $f(2) = 2$ and $f'(2) = 3$. Let $h(x) = \dfrac{f(x)}{x-3}$. Find $h'(2)$.

(b) Find constants a and b in the polynomial $p(x) = x^2 + ax + b$ such that $\lim_{x \to 4} \dfrac{p(x)}{x-4} = 3$.

[11] **5**. After studying sales for several months, the owner of a pizza chain knows at a price of \$14 per pizza, an average of 40 pizzas are sold per week, while at a price of \$17 per pizza, an average of 25 pizzas are sold per week. The fixed production costs are \$500 a week and it costs the owner \$8 to make a pizza. Let p be the unit price for the pizzas, and let q be the average number of pizzas sold per week. Assume that q is a linear function of the price p.

(a) Find the linear demand function $p = f(q)$.

(b) Find the cost function $C(q)$, the revenue function $R(q)$ and the profit function $P(q)$.

(c) What is the price the pizza chain owner should set for a pizza in order to maximize their weekly profit?

(d) Suppose that the price is currently $10 per pizza. If the price is decreased by a small amount, will the weekly revenue increase or decrease? (explain).

[5] **6**. Let

$$L = \lim_{h \to 0} \frac{(1+h)^{2018} + 2(1+h)^{2017} - 3}{h}.$$

(a) Find a constant c and two functions $f(x)$ and $g(x)$ such that $L = f'(c) + g'(c)$.

(b) Find $f'(-1) + \frac{1}{2} g'(-1)$, where $f(x)$ and $g(x)$ are the functions you found in the part (a) of Question 6.