The University of British Columbia

February 1, 2017

Common Midterm for All Sections of MATH 105 (Version 1)

Closed book examination Time: 60 minutes

Last Name ________________________ **First** ________________

Signature ________________

Student Number ________________

MATH 105 Section Number:__________

Special Instructions:

No memory aids are allowed. No calculators. No communication devices. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		12
2		8
3		8
4		9
5		9
6		4
Total		50

[12] 1. **Short-Answer Questions: Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.**

(a) [3] Find an equation of the plane that passes through the point $(-3, 2, -1)$ with a normal vector $\langle -3, 2, -1 \rangle$.

Answer:

(b) [3] Are the level curves of the function $z = 5x + 6y$ lines, planes or circles?

Answer:

(c) [3] Let $f(x,y) = e^{\sin(xy)}$. Find $\dfrac{\partial f}{\partial x}$.

Answer:

(d) [3] The following table shows values of a function $f(x,y)$ for values of x from 2.3 to 2.5 and values of y from 3.0 to 3.3.

$y \backslash x$	2.3	2.4	2.5
3.0	4.5	4.6	4.7
3.1	4.7	4.8	4.9
3.2	4.8	4.9	5.1
3.3	5.0	5.1	5.2

Use the table above to estimate the value of $f_x\ (2.4, 3.3)$.

Answer:

[8] **2.** Let $P : 2x + y - z = 3$ and $Q : x + y + z = 1$ be two planes.

(a) Determine if P and Q are orthogonal. You need to justify your answer to get the credits.

(b) Determine if P and Q are parallel. You need to justify your answer to get the credits.

(c) Is there a point in the yz-plane which is contained in both plane P and plane Q? You need to justify your answer to get the credits.

[8] **3**. Let $z = f(x, y) = \dfrac{1}{\sqrt{x^2 + 2y^2 - 5}}$.

(a) Find the domain of the function $z = f(x, y)$.

(b) Is the graph of the function $z = f(x, y)$ above the plane $z = -\dfrac{1}{2}$ or below the plane $z = -\dfrac{1}{2}$? Please give the reason for your answer.

(c) Find $\dfrac{\partial z}{\partial y}$.

(d) Find the rate of change in $\dfrac{\partial z}{\partial y}$ at $\left(\sqrt{7}, 1\right)$ as we change x but hold y fixed.

[9] 4. Let $f(x, y) = x^3 + y^3 - 6xy + A$, where A is a constant.

(a) Find all critical points of $f(x, y)$.

(b) Determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.

(c) Find the constant A such that -8 is a local minimum value of $f(x, y)$.

[9] **5**. Let R be the set $\{(x, y) : x^2 + y^2 \leq 10\}$.

(a) Use Lagrange multipliers to find the maximum and minimum values of $3x + y$ on the boundary of the set R. A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.

(b) Find the maximum and minimum value of $3x + y + 9$ on the set R.

[4] **6**. Determine if there exists a function $f(x, y)$ such that $f(x, y)$ has continuous partial derivatives of all orders, $f_{xy} = 2x^2 + 6y^2$, and $f_{xxx} = 3x^2 + 5y$. If your answer is YES, please find $f(x, y)$. If your answer is NO, please give your reason.

Solutions to Math 105 Midterm 1
(Feb. 1, 2017)

Version I

1. (a) $-3(x+3)+2(y-2)-(z+1)=0$ or $3x-2y+z=-14$.

(b) Lines.

(c) $\dfrac{\partial f}{\partial x} = e^{\sin(xy)} \cos(xy)\cdot y$.

(d) $f_x(2.4, 3.3) \approx \dfrac{f(2.5, 3.3) - f(2.4, 3.3)}{2.5-2.4} = \dfrac{5.2-5.1}{+0.1} = 1.$

or

$f_x(2.4, 3.3) \approx \dfrac{f(2.3, 3.3) - f(2.4, 3.3)}{2.3-2.4} = \dfrac{5.0-5.1}{-0.1} = 1$

2. We have $\vec{n}_P = \langle 2, 1, -1\rangle$ and $\vec{n}_Q = \langle 1, 1, 1\rangle$.

[2] (i) Since $\vec{n}_P \cdot \vec{n}_Q = \langle 2,1,-1\rangle \cdot \langle 1,1,1\rangle = 2+1-1 = 2 \neq 0$, P and Q are not orthogonal.

[2] (ii) Since $\langle 2, 1, -1\rangle \neq k\langle 1,1,1\rangle$ for any scalar k, P and Q are not parallel.

[4] (iii) Substituting $x=0$ into the equations of P and Q, we get $\begin{cases} y-z=3 \\ y+z=1 \end{cases}$, which gives $y=2$ and $z=-1$. Hence, the point $(0, 2, -1)$ in the yz-plane which is contained in both P and Q.

3. [1] (i) The domain is $\{(x,y): x^2+2y^2-5>0\}$.

[2] (ii) Above the plane $z=-\frac{1}{2}$ because $f(x,y)>0$ for all (x,y) in the domain.

[1] (iii) $$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left((x^2+2y^2-5)^{-1/2}\right) = -\frac{1}{2}(x^2+2y^2-5)^{-3/2}\cdot 4y$$

$$= -2y(x^2+2y^2-5)^{-3/2}.$$

[4] (iv) We need to find $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$ at $(\sqrt{7},1)$.

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(-2y(x^2+2y^2-5)^{-3/2}\right)$$

$$= -2y\left(-\frac{3}{2}\right)(x^2+2y^2-5)^{-5/2}\cdot 2x$$

$$= 6xy(x^2+2y^2-5)^{-5/2}$$

$$\frac{\partial^2 z}{\partial x \partial y}(\sqrt{7},1) = 6\sqrt{7}\,(7+2-5)^{-5/2} = 6\sqrt{7}\,(2^{-5})$$

4. [5] (i) $f_x = 3x^2 - 6y$, $f_y = 3y^2 - 6x$. We need to solve

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \quad \text{or} \quad \begin{cases} 3x^2 - 6y = 0 & (1) \\ 3y^2 - 6x = 0 & (2) \end{cases}$$

By (1), $y = \frac{1}{2}x^2$. Using this fact in (2), we get $3\left(\frac{1}{2}x^2\right)^2 - 6x = 0 \Rightarrow \frac{3}{4}x^4 - 6x = 0 \Rightarrow x = 0$ or $x = 2$.

$x = 0 \Rightarrow y = \frac{1}{2}x^2 = \frac{1}{2}\cdot 0^2 = 0$

$x = 2 \Rightarrow y = \frac{1}{2}x^2 = \frac{1}{2}\cdot 2^2 = 2$

Hence, $(0,0)$ and $(2,2)$ are all critical points

[2] (ii) $f_{xx} = 6x$, $f_{yy} = 6y$, $f_{xy} = -6$.

Critical points	f_{xx}	f_{yy}	f_{xy}	$D = f_{xx}f_{yy} - (f_{xy})^2$	Conclusion.
$(0,0)$	0	0	-6	$-$	saddle point
$(2,2)$	12	12	-6	$+$	local minimum

[2] (iii) By (ii), $f(2,2) = -8 \Rightarrow 2^3 + 2^3 - 6(2)(2) + A = -8$

$\Rightarrow A =$ [redacted]. i.e. $A = 0$

5. (i) Let $f(x,y)=3x+y$, $g(x,y)=x^2+y^2-10$. We need
[6] to solve the system $\begin{cases} f_x=\lambda g_x \\ f_y=\lambda g_y \\ g=0 \end{cases}$ or

$$\begin{cases} 3=2\lambda x & (1) \\ 1=2\lambda y & (2) \\ x^2+y^2-10=0 & (3) \end{cases}$$

By (1) and (2), $\frac{3}{1}=\frac{2\lambda x}{2\lambda y}=\frac{x}{y} \Rightarrow x=3y \quad (4)$

Using (4), (3) becomes $(3y)^2+y^2-10=0 \Rightarrow y=\pm 1$.

Then f has possible extreme values at $(3,1)$ and $(-3,-1)$. We compute $f(3,1)=10$ and $f(-3,-1)=-10$, so the maximum value of f on $x^2+y^2=10$ is $f(3,1)=10$, and the minimum value is $f(-3,-1)=-10$.

[3] (ii) Since $f_x=3$ and $f_y=1$ for $f(x,y)=3x+y$, $f(x,y)=3x+y$ does not have a critical point on the set R. Hence, 10 is the maximum value of $f(x,y)$ on the set R, and -10 is the minimum value of $f(x,y)$ on the set R. It follows that $10+9=19$ is the maximum value of $3x+y+9$ on R, and $-10+9=-1$ is the minimum value of $3x+y+9$ on R.

6. $f_{xyx} = \frac{\partial}{\partial x}(2x^2 + 6y^2) = 4x,$

$f_{xyxx} = \frac{\partial}{\partial x}(4x) = 4.$

$f_{xxxy} = \frac{\partial}{\partial y}(3x^2 + 5y) = 5.$

Since $f_{xyxx} \neq f_{xxxy}$, there does not exist the function with the properties.