Solutions to Math 184 Midterm 2 (Nov. 15, 2018)

(Version I)

1. (a) $\frac{dy}{dx} = 4(\cos^3 x)(-\sin x) - (\sin(x^4))\,4x^3$.

$\boxed{\text{Answer: } -4(\cos^3 x)\sin x - 4x^3\sin(x^4)}$

(b) $f'(x) = \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2$

$\boxed{\text{Answer: } \frac{1}{\ln(2x)} \cdot \frac{1}{2x} \cdot 2 \left(= \frac{1}{x\ln(2x)}\right)}$

(c) $f'(x) = \frac{3}{8}x^2 - \frac{1}{2}$

$f' = 0 \Rightarrow \frac{3}{8}x^2 - \frac{1}{2} = 0 \Rightarrow x = \pm\frac{2}{\sqrt{3}}$

$\boxed{\text{Answer: } \pm\frac{2}{\sqrt{3}} \left(= \pm\sqrt{\frac{4}{3}}\right)}$

(d) $f(x) = \frac{2x(x^2-x-6)}{x^2(x+3)} = \frac{2(x-3)(x+2)}{x(x+3)}$

$\boxed{\text{Answer 1: } x=0,\ x=-3}$

$\lim_{x\to 0^+} f(x) = -\infty \Rightarrow x=0$ is a VA,

$\lim_{x\to -3^+} f(x) = -\infty \Rightarrow x=-3$ is a VA.

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2-\frac{2}{x}-\frac{6}{x^2}}{1+(3/x)} = 2 \Rightarrow y=2$ is a HA.

$\boxed{\text{Answer 2: } y=2}$

(e) $f' = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\cdot\sqrt[3]{x}} \Rightarrow x=0$ is the unique critical point.

$f(0)=0$, $f(-8) = (-8)^{\frac{2}{3}} = 2^2 = 4$,

$f(1) = 1^{2/3} = 1$.

$\boxed{\text{Answer: } A=4,\ B=0}$

2. Let V be the volume of the sand pile, let r be the radius of the sand pile, and let h be the height of the sand pile.

Then $r = 4h$. Hence, we get

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (4h)^2 h = \frac{16}{3}\pi h^3$$

$$\frac{dV}{dt} = \frac{16}{3}\pi \cdot 3h^2 \cdot \frac{dh}{dt} = 16\pi h^2 \frac{dh}{dt}$$

At the instant that $h = 8$, we get

$$24 = \frac{dV}{dt} = 16\pi \cdot 8^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{24}{16\pi(8^2)} \ (ft/min).$$

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3. (a) [5] $\frac{d}{dx}\left(3(x^2+y^2)^2\right) = \frac{d}{dx}\left(25(x^2-y^2)\right)$

$$3\cdot 2(x^2+y^2)(2x+2yy') = 25(2x-2yy')$$

$$6x(x^2+y^2)+6(x^2+y^2)yy' = 25x - 25yy'$$

$$(6x^2+6y^2+25)yy' = (25-6x^2-6y^2)x$$

$$y' = \frac{(25-6x^2-6y^2)x}{(6x^2+6y^2+25)y}$$

At $x=2$, we have $y=1$. Hence, the slope of the tangent line is given by $y'(2) = \frac{(25-24-6)\cdot 2}{(24+6+25)\cdot 1} = -\frac{2}{11}$.

(b) [4] $\ln f(x) = \ln(\sin x)^{\cos(3x)} = \cos(3x)\ln(\sin x)$

$$\frac{d}{dx}\left(\ln f(x)\right) = \frac{d}{dx}\left(\cos(3x)\cdot\ln(\sin x)\right)$$

$$\frac{f'(x)}{f(x)} = -\sin(3x)\cdot 3\cdot\ln(\sin x) + \cos(3x)\cdot\frac{1}{\sin x}\cdot\cos x$$

$$f'(x) = f(x)\left[-3(\sin(3x))\ln(\sin x) + \frac{(\cos(3x))\cos x}{\sin x}\right]$$

$$\left(= (\sin x)^{\cos(3x)}\left[-3(\sin(3x))\ln(\sin x) + \frac{(\cos(3x))\cos x}{\sin x}\right]\right)$$

4. (a) [2]

$$\frac{dq}{dp} = 10e^{1-p}(-1) - \frac{5}{(p+1)^2} = -10e^{1-p} - \frac{5}{(p+1)^2}$$

$$E(p) = \frac{p}{q}\frac{dq}{dp} = \frac{p}{q}\left(-10e^{1-p} - \frac{5}{(p+1)^2}\right)$$

$$\left(= \frac{p}{\left(10e^{1-p} + \frac{5}{p+1}\right)} \cdot \left(-10e^{1-p} - \frac{5}{(p+1)^2}\right)\right)$$

(b) [3] If $p=1$, then $q = 10 + \frac{5}{2}$ and

$$E(1) = \frac{1}{10+\frac{5}{2}}\left(-10 - \frac{5}{4}\right) > -1.$$

So revenue increases.

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5. (a)[4] $f'(x) = \left((\ln x)\sqrt{x}\right)' = \frac{1}{x}\sqrt{x} + (\ln x)\cdot\frac{1}{2\sqrt{x}}$

$$= \frac{2+\ln x}{2\sqrt{x}}.$$

$f'(x)=0 \Rightarrow 2+\ln x = 0 \Rightarrow x = e^{-2}$ is a critical point.

f': 0 $-$ e^{-2} $+$

f: $\searrow$ $\nearrow$

Hence, $f(x)\uparrow$ on (e^{-2}, ∞), and

$f(x)\downarrow$ on $(0, e^{-2})$.

(b)[3] $f'' = \left(\frac{2+\ln x}{2\sqrt{x}}\right)' = \frac{1}{2}\left(\frac{2+\ln x}{\sqrt{x}}\right)' = \frac{1}{2}\cdot\frac{\frac{1}{x}\sqrt{x} - (2+\ln x)\cdot\frac{1}{2\sqrt{x}}}{x}$

$$= \frac{1}{2}\cdot\frac{2-(2+\ln x)}{2x\sqrt{x}} = -\frac{\ln x}{4x\sqrt{x}}.$$

$f''=0 \Rightarrow \ln x = 0 \Rightarrow x = 1.$

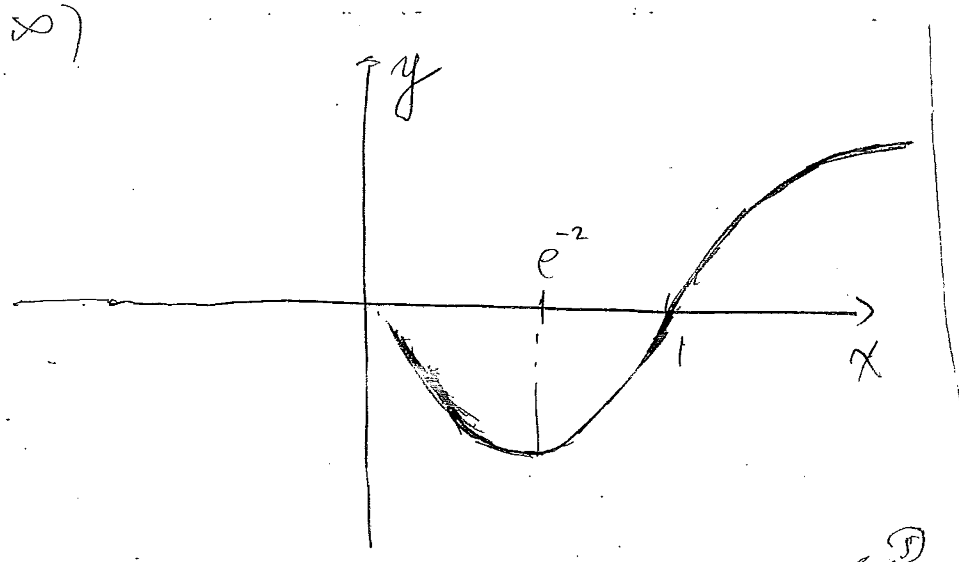
f'': 0 $+$ 1 $-$

f: CU CD

f is CU on $(0,1)$, and

f is CW on $(1, \infty)$

(c)[3]



6.

$$F(t) = 6000\, e^{rt}$$

$$7260 = F(2) = 6000\, e^{2r} \Rightarrow e^{2r} = \frac{7260}{6000} = \frac{726}{600}$$

$$\Rightarrow r = \frac{1}{2}\ln\left(\frac{726}{600}\right)$$

$$E(t) = (7260 - 4260)\, e^{t\left(\frac{1}{2}\ln\left(\frac{726}{600}\right)\right)}$$

$$E(1) = 3000\, e^{\frac{1}{2}\ln\left(\frac{726}{600}\right)} \quad \left(= 3000\sqrt{\frac{726}{600}}\right).$$

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