The University of British Columbia

November 15, 2018

Common Midterm 2 for Sections of MATH 184 (Version 1)

Closed book examination Time: 60 minutes

Last Name _____ **First** _____

Signature _____

Student Number _____

MATH 184 Section Number: _____

Special Instructions:

No memory aids are allowed. No calculators. No communication devices. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		15
2		5
3		9
4		5
5		10
6		6
Total		50

Short-Answer Questions: Put your answer in the box provided. Full marks will be given for a correct answer placed in the box, while part marks may be given for work shown. Unless otherwise stated, calculator ready answers are acceptable.

[15] **1.**

(a) Find $\frac{dy}{dx}$, where $y = \cos^4 x + \cos(x^4)$.

Answer:

(b) Find $f'(x)$, where $f(x) = \ln\big(\ln(2x)\big)$.

Answer:

(c) Find the critical point(s) of $f(x) = \frac{1}{8}x^3 - \frac{1}{2}x$.

Answer:

(d) Find the vertical and horizontal asymptotes of $f(x) = \frac{2x^3 - 2x^2 - 6x}{x^3 + 3x^2}$. Please write the vertical asymptote(s) in Answer 1, and write the horizontal asymptote(s) in Answer 2.

Answer 1:

Answer 2:

(e) Let A be the absolute maximum value of $f(x) = x^{\frac{2}{3}}$ on the interval $[-8, 1]$, and let B be the absolute minimum value of $f(x) = x^{\frac{2}{3}}$ on the interval $[-8, 1]$. Find A and B.

Answer:
$A =$, $B =$

Full-Solution Problems: Justify your answers and show your work. Place a box around your final answer. Unless otherwise indicated, simplification of answers is required in these questions.

[5] **2.** Sand falls from an overhead bin, accumulating in a conical pile with a radius that is always four times its height. If the sand falls from the bin at a rate of 24 ft^3/min, how fast is the height of the sandpile changing when the pile is 8 ft high? (Hint: the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.)

[9] **3.**

(a) Find the slope of the tangent line to the curve defined by

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(2, 1)$.

(b) Consider the function $f(x) = (\sin x)^{\cos(3x)}$, defined on $\left(0, \frac{\pi}{2}\right)$. Find the derivative $f'(x)$ of $f(x)$.

[5] **4.** Assume that the price p and the demand q of a certain good are related by the equation

$$q = 10e^{1-p} + \frac{5}{p+1}.$$

(a) Find the price elasticity $E(p)$ of demand in terms of p and q.

(b) If $p = 1$, does increasing price slightly increase revenue?

[10] **5.** Let $f(x) = (\ln x)\sqrt{x}$.

(a) Find the intervals on which $f(x)$ is increasing or decreasing.

(b) Find the intervals on which $f(x)$ is concave up or concave down.

(c) Sketch the graph of $f(x)$.

[6] **6.** The student A borrowed \$6000 from the student B who had taken Math 184, and who therefore had a preference for continuous compounding. Two years later the student B informed the student A that his debt had grown to \$7260. So the student A paid the student B \$4260, and paid off the rest of the debt one year later. What was that final payment?