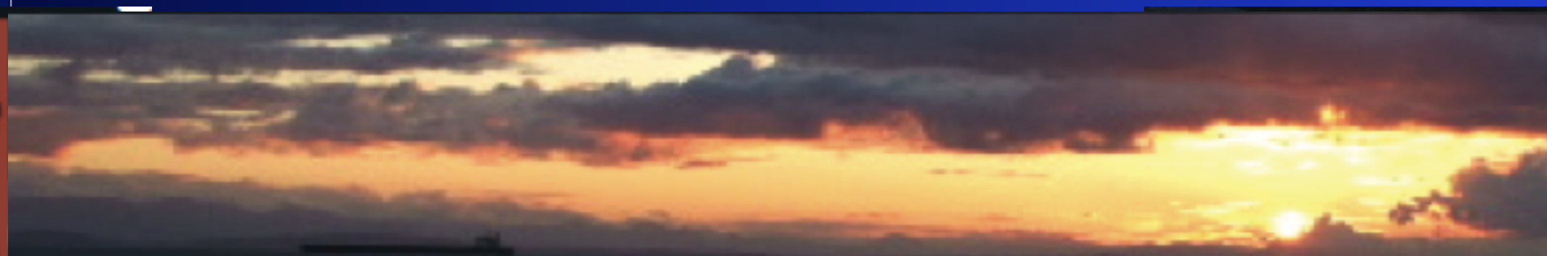
Mathematical Cell Biology Graduate Summer Course

University of British Columbia, May 1-31, 2012

Leah Edelstein-Keshet

Simple biochemical motifs (1)

mprime



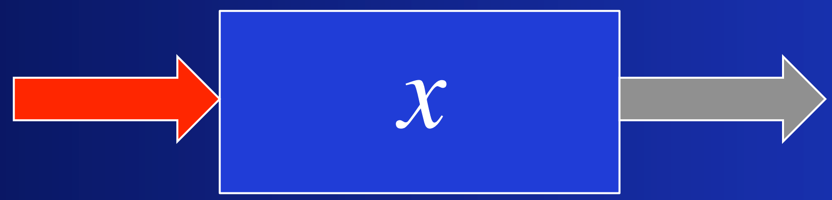
PIMS

www.math.ubc.ca/~keshet/MCB2012/

Biochemical (and gene) circuits

Switches, oscillators, adaptation, and amplification circuits

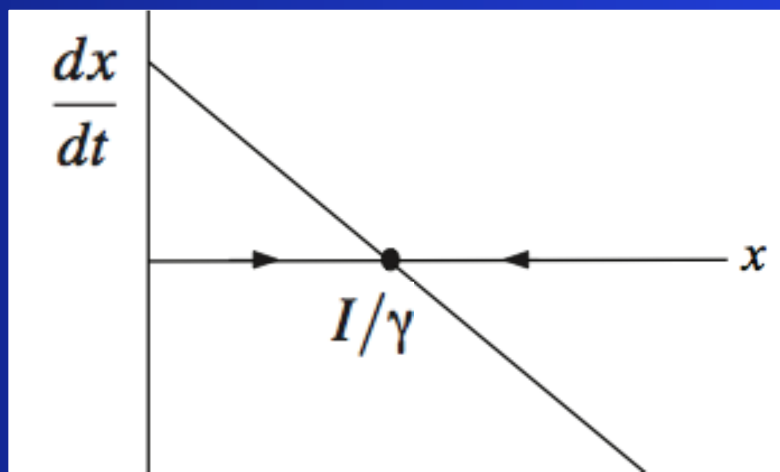
Production-decay at constant rates



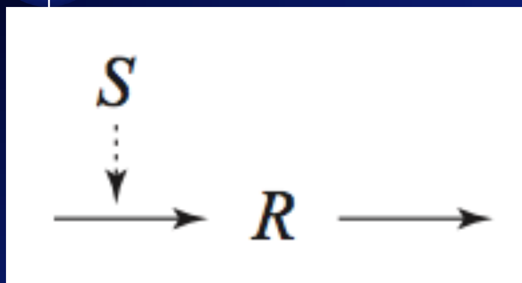
$$\frac{dx}{dt} = I - \gamma x$$

$I, \gamma > 0$ constants.

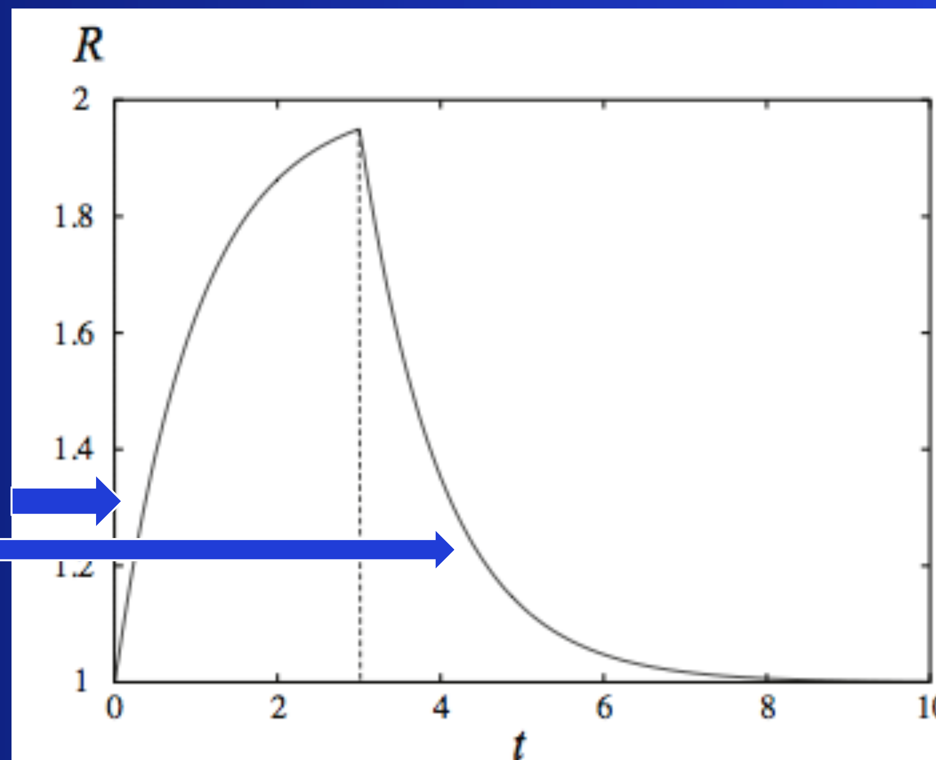
Unique positive Steady state



Signal-induced Production

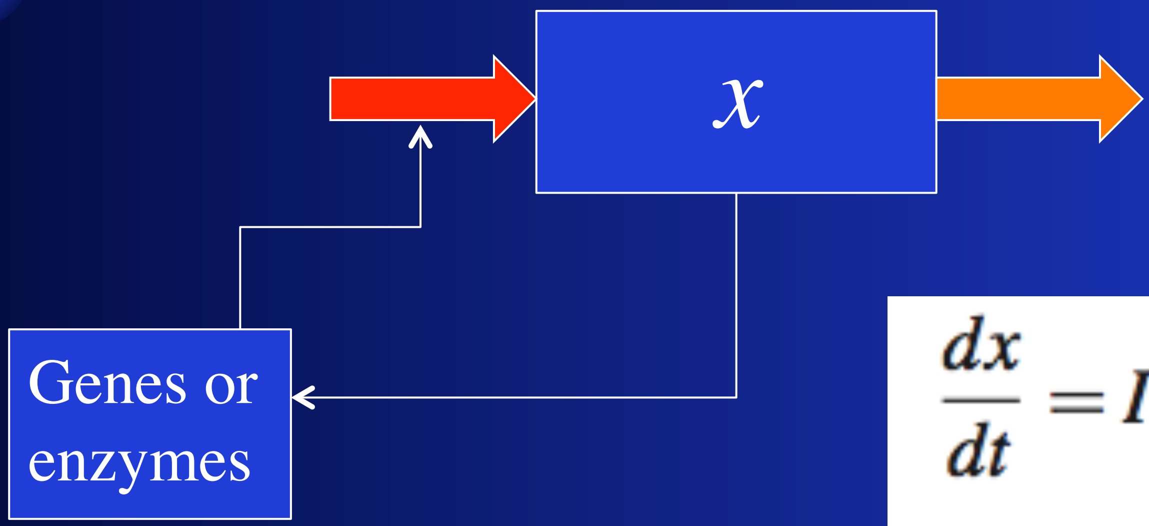


$$\frac{dR}{dt} = k_0 + k_1 S - k_2 R.$$



Note typical "1-exp(-k_2t)" rise and exponential decay tail

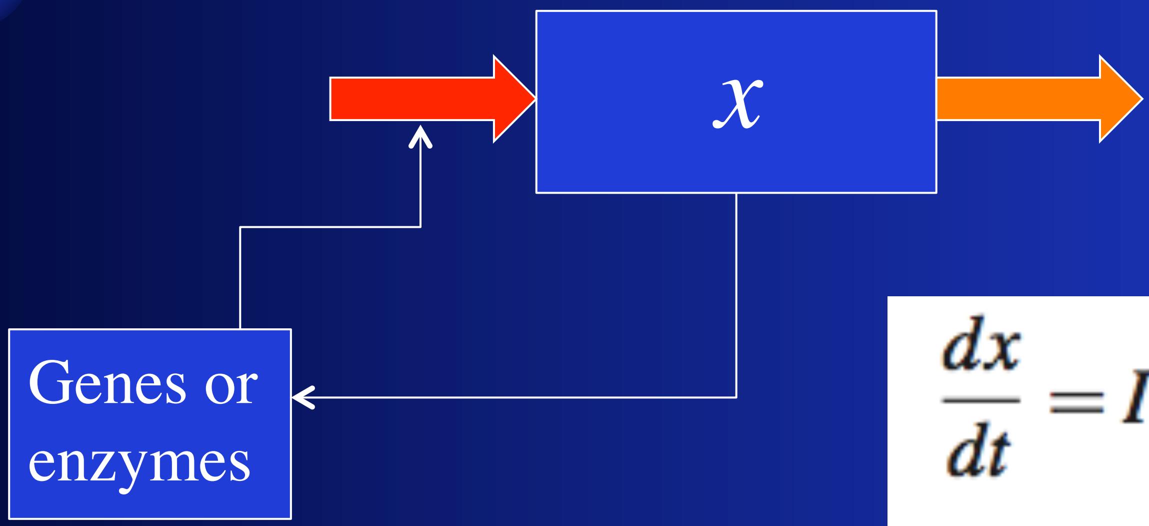
Feedback to production



$$\frac{dx}{dt} = I - \gamma x$$

I is now a function of *x*

Michaelian Feedback to production



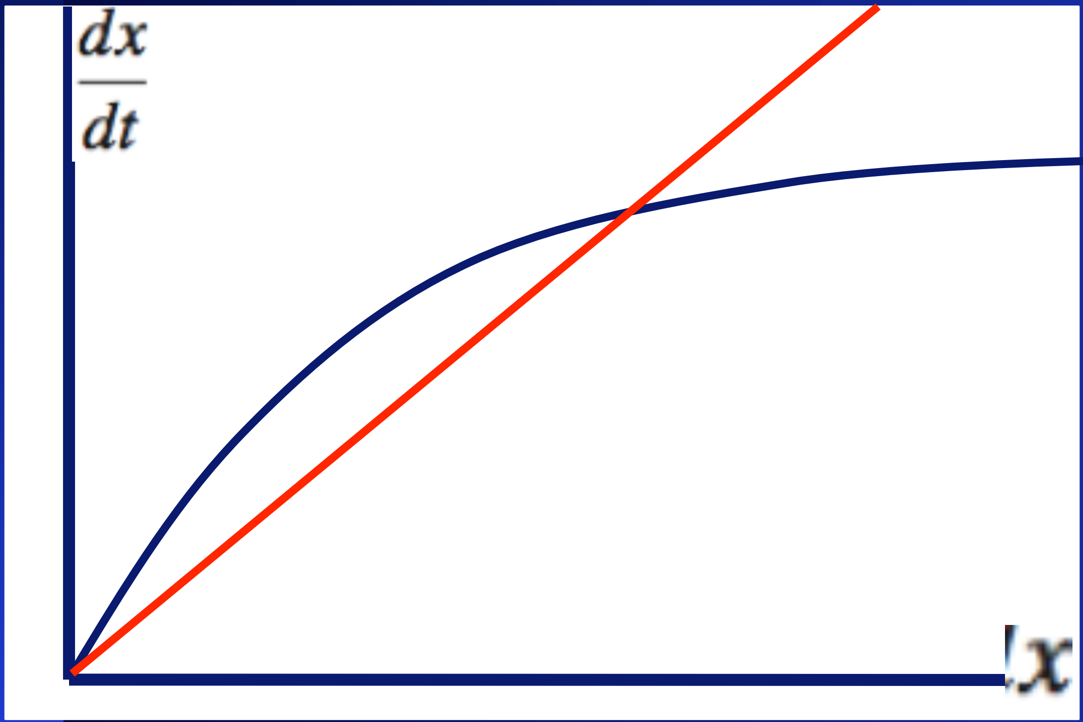
$$\frac{dx}{dt} = I - \gamma x$$

$$I(x) = I_0 + \frac{I_{max}\; x}{k_n + x}$$

Michaelian Feedback to production

$$I(x) = I_0 + \frac{I_{max}\ x}{k_n + x}$$

$$\frac{dx}{dt} = I - \gamma x$$

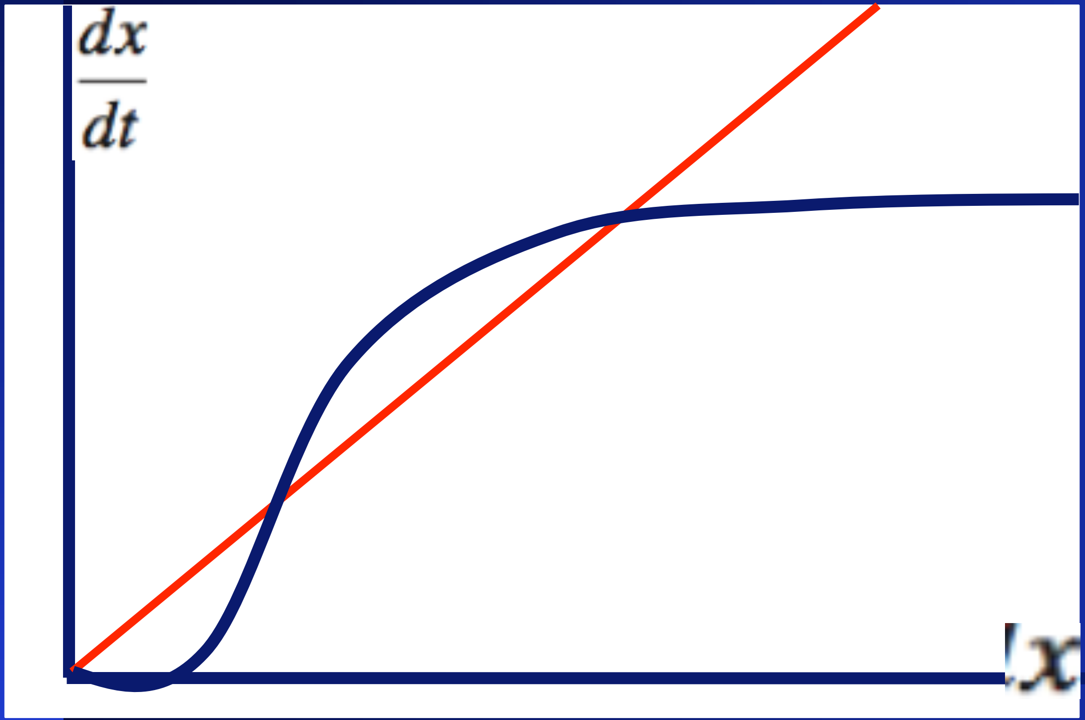


At most 2 steady states, one stable.

Sigmoidal Feedback to production

$$I(x) = I_0 + \frac{I_{max}\ x^2}{k_n^2 + x^2}$$

$$\frac{dx}{dt} = I - \gamma x$$



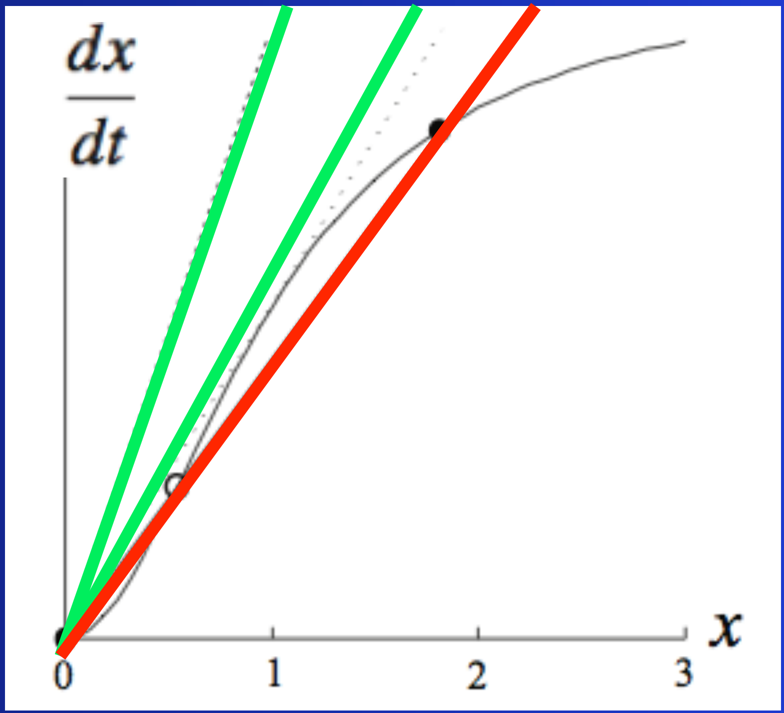
Up to 3 steady states, two stable.

"bistability"

Sigmoidal cont'd

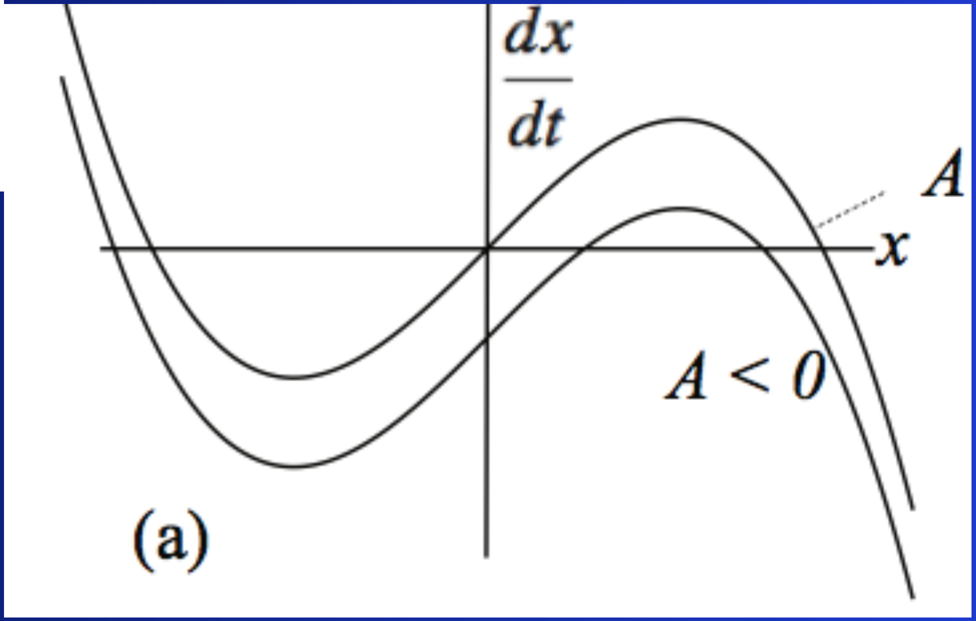
$$\frac{dx}{dt} = f(x) = \frac{x^2}{1+x^2} - mx + b$$

Actual number of steady states depends on parameters, e.g. on slope m (decay rate of x)

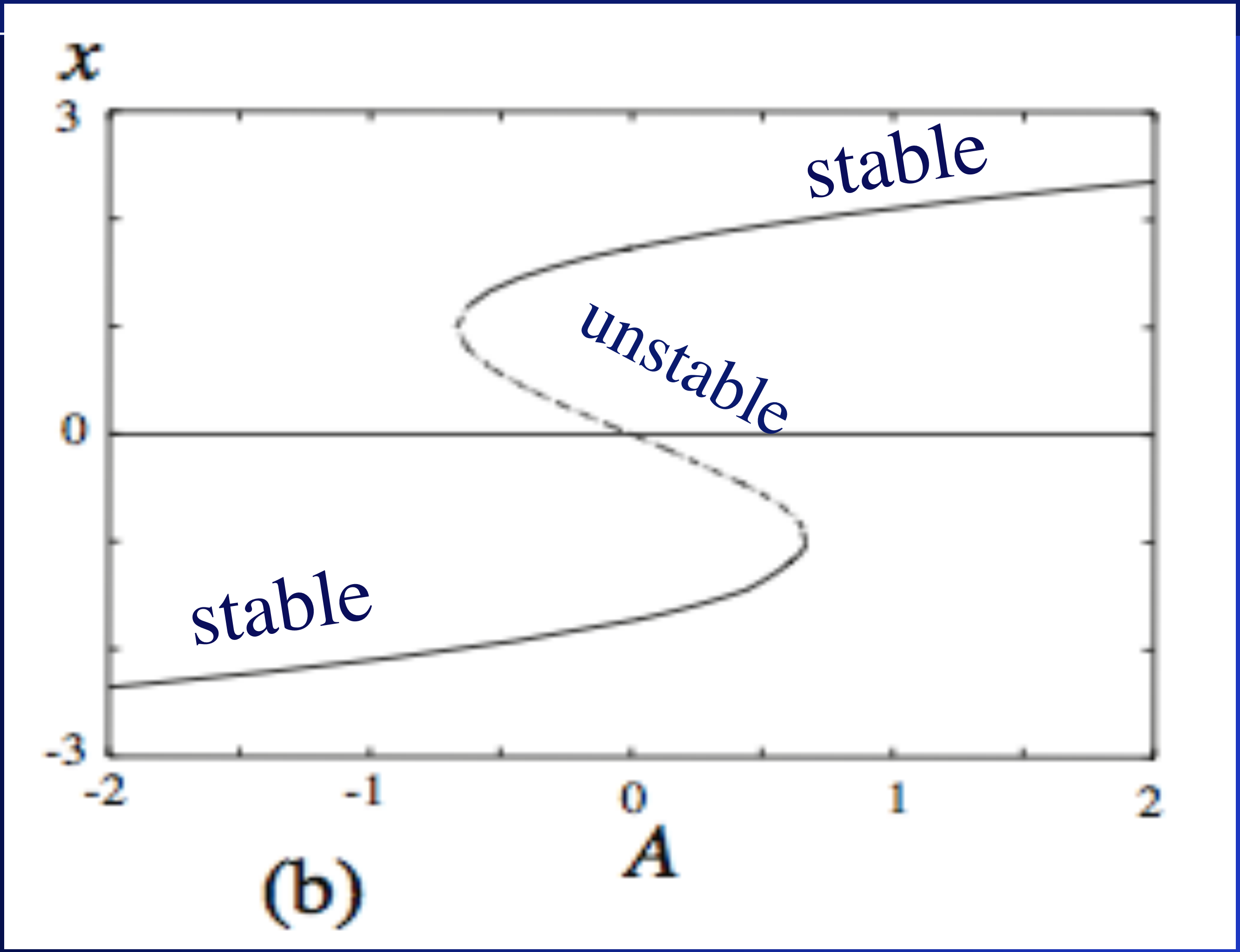


Generic bistability

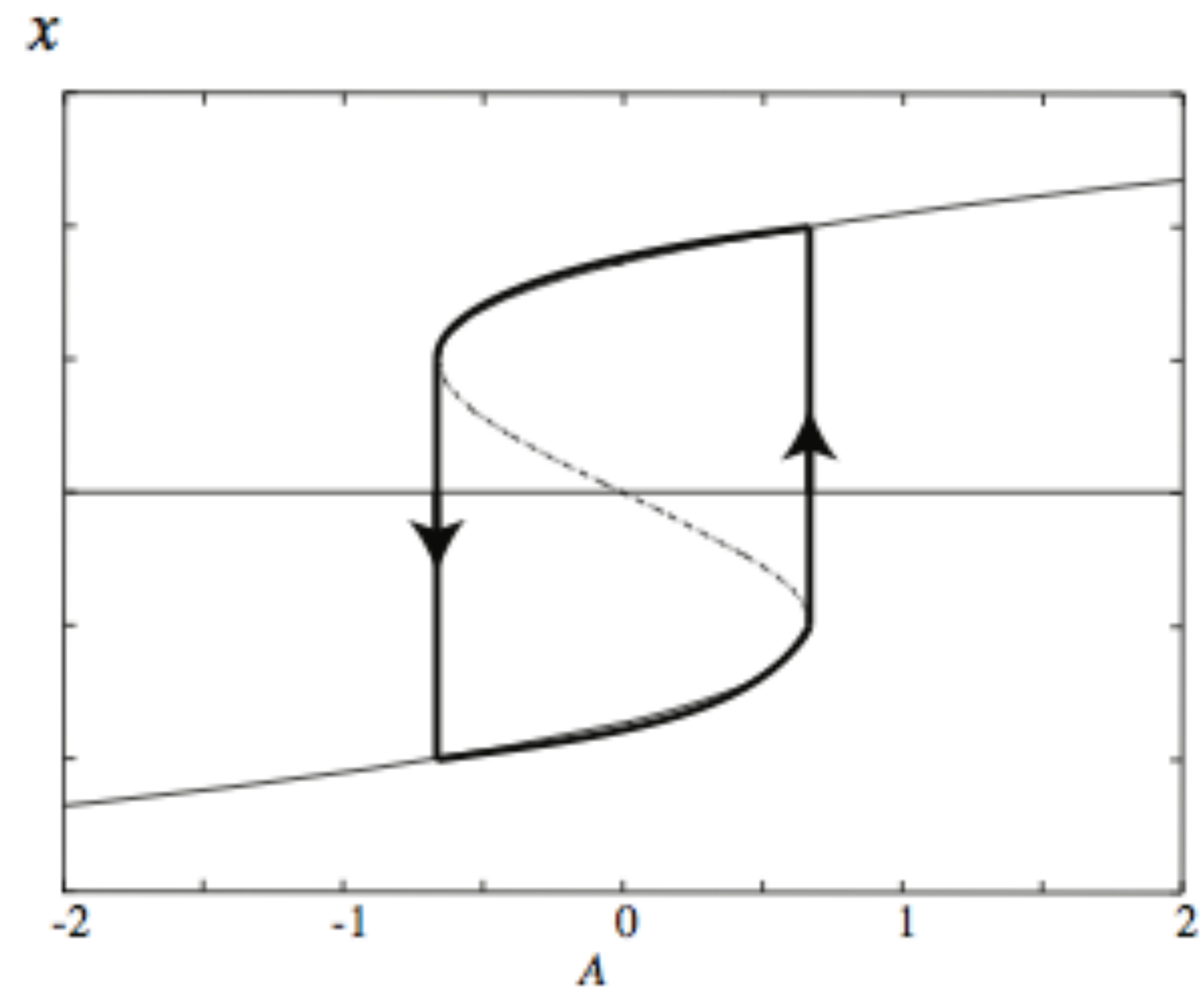
$$\frac{dx}{dt} = c\left(x - \frac{1}{3}x^3 + A\right)$$



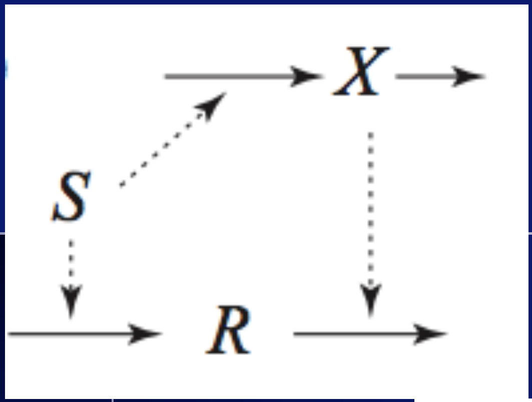
Bifurcation Diagram



Hysteresis



Adaptation



$$\frac{dR}{dt} = k_1 S - k_2 X R,$$
$$\frac{dX}{dt} = k_3 S - k_4 X.$$