The University of British Columbia

Department of Mathematics

MATHEMATICS COLLOQUIUM

The Kervaire Invariant: What we still don't know about the real projective plane

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Time: **3:00 p.m.**
Location: **MATH ANNEX 1100**

Let $S^m = \{(x_0, x_1, \ldots, x_m) \in \mathbb{R}^{m+1} \mid x_0^2 + x_1^2 + \ldots + x_m^2 = 1\}$ be the m-dimensional sphere. Given $m \geq n \geq 1$, a primary concern of topologists is to classify the (continuous) maps from S^m to S^n up to homotopy, where two maps f, g are said to be homotopic iff f can be continuously deformed into g. In the well-known case when $m = n = 1$, classification is attained by associating to $f : S^1 \to S^1$ a *winding number* $\omega(f) \in \mathbb{Z}$, which is invariant when f undergoes continuous deformation.

The *Kervaire Invariant* $\alpha(f) \in \mathbb{Z}/2\mathbb{Z}$ of a map $f : S^m \to S^n$, definable via framed cobordism for some values of m and n with $m > n$, is an invariant much more subtle than the winding number. For the past 40 odd years, the search for maps f with $\alpha(f) \neq 0$ continues to occupy a central stage in homotopy theory and differential topology. In this talk I shall explain what the Kervaire invariant is, and show how it can emerge from the study of certain questions intrinsic to the geometry of the real projective plane. Some work in progress will be described, while technical details will be kept to a minimum.

Refreshments will be served at 2:45 p.m. in the Faculty Lounge, Math Annex (Room 1115).