Math 101 - 951 Quiz #2 (July 16, 2014)

Show all your work. Use back of page if necessary. Calculators are not allowed.

Last Name: **First Name:** **Student No.:**

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1. (6 marks, 3 for each part below) Evaluate the following integrals:

   (a) $\int 2t\,(\ln t)^2\,dt$

   Solution: Integrate by parts by identifying $f(t) = (\ln t)^2$, $g'(t) = 2t$, $f'(t) = \frac{2}{t}\ln t$, $g(t) = t^2$:

   $$\int 2t(\ln t)^2\,dt = t^2(\ln t)^2 - \int 2t\ln t\,dt.\text{①}$$

   Integrate by parts again, with $f(t) = \ln t$, $g'(t) = 2t$, $f'(t) = 1/t$, $g(t) = t^2$:

   $$\int 2t(\ln t)^2\,dt = t^2(\ln t)^2 - t^2\ln t + \int t\,dt\text{①} = t^2(\ln t)^2 - t^2\ln t + \frac{t^2}{2} + C.\text{①}$$

   (No need to penalize omission of $+C$.)

   (b) $\int_0^{\frac{\pi}{2}} (\cos x)^{\frac{7}{3}} \sin^3 x\,dx$

   Solution: Since $\sin x$ appears in odd power, we can make the substitution $u = \cos x$, $du = -\sin x\,dx$ (note the change of the lower and upper bounds into those for u):

   $$\int_0^{\frac{\pi}{2}} (\cos x)^{\frac{7}{3}} \sin^3 x\,dx = -\int_1^0 u^{\frac{7}{3}}(1-u^2)\,du\text{①} = \int_0^1 (u^{\frac{7}{3}} - u^{\frac{13}{3}})\,du = \left[\frac{3}{10}u^{\frac{10}{3}} - \frac{3}{16}u^{\frac{16}{3}}\right]_0^1 \text{①} = \frac{9}{80}.\text{①}$$

   Deduct ① for incorrect bounds.

2. (4 marks) Find the volume of the solid obtained by rotating the region enclosed by the curves $y = 1 - x^2$ and $y = |x| - 1$ about the line $y = -1$.

   Solution: A sketch shows that we have an upside-down parabola $y = 1 - x^2$ over a corner formed by two straight lines $y = \pm x - 1$ emanating from $(0, -1)$. They intersect at $(-1, 0)$ and $(1, 0)$, and are symmetric about the y axis. The volume is thus twice that computed from the right half of the graph alone (award ① for sketch and correct bounds):

   $$V = 2\pi\int_0^1 \left[(1-x^2+1)^2 - (x-1+1)^2\right]dx\text{①} = 2\pi\int_0^1 (x^4 - 5x^2 + 4)\,dx = 2\pi\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1 \text{①} = \frac{36\pi}{5}.\text{①}$$