Math 101 - 951 Quiz #1 (July 9, 2014)

Show all your work. Use back of page if necessary. Calculators are not allowed.

Last Name: **First Name:** **Student No.:**

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1. (6 marks, 3 for each part below) Evaluate the following integrals:

   (a) $\int x\sin(x^2)\cos(x^2)dx$

   Solution: Taking intermediate variable $u(x) = x^2$ and $du = 2x\,dx$:

   $$\int x\sin(x^2)\cos(x^2)dx = \frac{1}{2}\int \sin u\cos u\,du.\textcircled{1}$$

   Taking $v(u) = \sin u$, $dv = \cos u\,du$:

   $$\int x\sin(x^2)\cos(x^2)dx = \frac{1}{2}\int v\,dv\textcircled{1} = \frac{1}{4}v^2 + C = \frac{1}{4}(\sin x^2)^2 + C.\textcircled{1}$$

   (No need to penalize omission of $+C$.)

   (b) $\int_0^2 |t^2 - 1|\,dt$

   Solution: Since the integrant function has two segments, we evaluate the integral in two intervals:

   $$\int_0^2 |t^2-1|\,dt = \int_0^1 (1-t^2)\,dt + \int_1^2 (t^2-1)\,dt\textcircled{1} = \left[t - \frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3} - t\right]_1^2 \textcircled{1} = \frac{2}{3} + \frac{4}{3} = 2.\textcircled{1}$$

2. (4 marks) Use the first part of the Fundamental Theorem of Calculus (FTC1) to evaluate the limit $\lim_{h\to 0}\frac{1}{h}\int_0^h \sqrt{e^t - 1}\,dt$.

   Solution: If we define function $g(x) = \int_a^x \sqrt{e^t-1}\,dt$ $\textcircled{1}$, we recognize the limit as the derivative

   $$g'(0) = \lim_{h\to 0}\frac{g(h) - g(0)}{h} = \lim_{h\to 0}\frac{1}{h}\int_0^h \sqrt{e^t-1}\,dt.\textcircled{1}$$

   From FTC1, we have $g'(x) = \sqrt{e^x - 1}$ $\textcircled{1}$, and finally $g'(0) = \sqrt{e^0 - 1} = 0.$ $\textcircled{1}$