Outline for final review (Math 101-951, August 2014)

Disclaimer: a summary of key points to aid your review. Not meant to be exhaustive and no guarantee that it covers all test problems. Blue text indicates key supporting material from earlier studies.

Three chunks of material:

I. Techniques for integration: how to find antiderivatives of certain functions.

   a. Substitution: recognize patterns (see midterm review)
   b. Integration by parts: recognize familiar patterns (e.g. $x \sin(x)$, xe^x, $x \ln x$)
   c. Trigonometric functions: rules for $\cos^m \sin^n$, $\tan^m \sec^n$; different odd/even combinations (trigonometric identities; completing squares under sq. root).
   d. Trigonometric substitution: $\sqrt{a^2 \pm x^2}$; draw right triangle to transform result from θ to x (trigonometric identities).
   e. Partial fractions for rational functions: cases 1, 2, 3 (factorizing polynomials).
   f. Strategies for integration

II. Applications of integration

   a. Area under/between curves; volume of solids of revolution
   b. Work (taking slice and using geometry to figure out its area/vol./weight)
   c. Fundamental theorem of calculus (often requires chain-rule in differentiation)
   d. Approximate integration
      i. Riemann sums L_n and R_n
      ii. Midpoint, trapezoid and Simpson's rules (needn't memorize error formulas)
   e. Center of mass: remember formulas; mind upper and lower curves
   f. Separable equations: mixing problems (often involves *ln* and exponential manipulation)

III. Sequences & series (+ improper integrals)

   a. Much of the following involves limits: review prior limit-finding techniques, e.g. l'Hopital's rule etc.
   b. Improper integrals: definition of the 2 types; comparison test
   c. Sequence: connection to functions on the one hand, to series on the other
      i. Limit of sequence $\leftrightarrow$ limit of function: $a_n = f(n)$
      ii. Connection and difference between sequence and series: partial sum forms a sequence; its convergence $\leftrightarrow$ convergence of series
      iii. Squeeze theorem (follows from counterpart for functions)
      iv. Monotonic sequence theorem

d. Series: how to test convergence/divergence?
   i. Most general:
      1. Ratio test for absolute convergence: appreciate "pecking order" of common functional forms (involves limits)
      2. Divergence test: a_n tending to zero as *necessary* (but *insufficient*) condition for convergence of series
   ii. Testing positive series
      1. Integral test (connection to Improper Integrals)
      2. Comparison test (algebraic manipulation to find inequality)
      3. Limit comparison test: recognize leading order terms (involves limit)
      4. The last 2 tests often use p-series ($\sum \frac{1}{n^p}$) and geometric series ($\sum ar^{n-1}$)
   iii. Alternating series: test of 2 conditions – decreasing and limit
   iv. Power series $\sum c_n x^n$ and $\sum c_n (x-a)^n$:
      1. Ratio test to determine radius of convergence R; check end points to determine interval of convergence I.
      2. Differentiation and integration of series by term does not change R.

e. Series: how to evaluate series?
   i. Sum of geometric series: learn to recognize them: $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$
   ii. Sum estimation of alternating series: $|R_n| \leq b_{n+1}$

f. Representing functions using power series
   i. Series expression for common functions obtained by algebraic manipulation, differentiation and integration, e.g. $ln(1 \pm x)$.
   ii. Taylor/Maclaurin series for common functions: $sin(x)$, e^x, etc.
   iii. Use Taylor/Maclaurin series to approximate integrals; relates to item d-ii above.