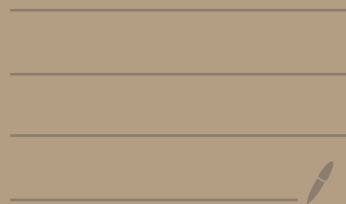


Solutions to HW1 Problem 3



We wish to stratify $\overline{M}_1(\mathbb{P}^1, 2[\mathbb{P}^1])$ by topological type and identify each stratum. Each stratum is determined by its labelled dual graph which consists of a connected graph whose vertices v_i are labelled by (g_i, d_i) the genus and the degree of the corresponding component. We need the labelled graph to satisfy:

- The total degree is 2: $\sum d_i = 2$
- The genus is 1: $\# \text{ of cycles in graph} + \sum g_i = 1$
- There are no vertices with $(g, d) = (1, 1)$ (no degree 1 map of genus 1 curve to \mathbb{P}^1)
- Stability: any vertex labelled with $(0, 0)$ must have valence ≥ 3 .
- Also: a $(0, 1)$ node cannot have a loop (since the normalization of the corresponding component is an isomorphism and hence the two points over the node must go to distinct points).

Using the above, one can list all possible graphs. We list them in a table on the next page along with the dimension of the strata and a picture of the map

(we draw branched covers as  whereas nodes are depicted by crossings )

Graph labelled by (g_i, d_i)	Map	dim	Graph labelled by (g_i, d_i)	Map	dim
		4			2
		3			2
		4			2
		3			1
		2			1
		3			
		3			