Klaus Hoechsmann, Jan. 22, 1996

Higher math for elementary school teachers.

Rationale. Math335 at UBC is a low-level, high-intensity introduction to mathematics for students intending to enrol in the "elementary" programme of the Faculty of Education. They could have satisfied the faculty's minimal mathematics requirement by taking any normal introductory math course, but those who queue up for Math335 have usually waited till the very last moment for their plunge into the subject which is, after all, the only science commonly associated with the word "anxiety".

Mathemaphobia, popularly known as math anxiety, is not a purely academic concern: it obstructs access not only to mathematics itself but to a whole range of activities, from econometrics to engineering, seen as "too technical". Thus it deprives our increasingly technical societies of human resources, both in creative energy and informed political judgment, while undermining its victims' self-confidence and contributing to their sense of alienation. In some cases, it may be blocking a person's one major talent. However, like most phobias, it often masquerades as disinterest or disdain.

Since school seems to be a fertile breeding ground for this virus, a modest step toward its elimination would be to contain its spread among teachers. This cannot be accomplished through good-will campaigns or intensive rote training. Teachers, especially in elementary school, need to move about their subjects with a certain ease and spontaneity: they must be seen as people who can live what they teach. A few brave slogans and formulas, grafted on a stem of ignorance and fear, will not convince the children that mathematics is worth learning. They need to see its human face.

There seems to be no great problem in the early grades: coloured rods, geoboards, and the like help to make math a favorite subject among the youngest children and their teachers. Leaving aside the conundrum of secondary school, remedial efforts would probably have their most beneficial effect on the crucial second half of the elementary cycle. There, in the context of fractional quantities, teachers usually come to the end of their tether[1], and are reduced to enforcing the laws proclaimed by the text-book.

As mathematicians, we are grateful for the opportunity of some input into the school system. Some of us may bemoan the depth of "innumeracy" we find in the students of Math335 and be tempted give up on them. However, we had better recognize that many of them, though mathematically well below university standards, are actually quite bright, and — like it or not — most of them will probably wind up in schools as teachers of our progeny. We have a small (3 months) window of opportunity to influence their attitude toward mathematics and, beyond that, other things technical.

Syllabus. Math335 is supposed to be neither a course in teaching arithmetic, nor a course *about* mathematics, but an excursion into the subject itself. In designing such a course, it is important to strike a balance between puzzles and problems on the one hand, and coherent theory on the other. For relieving math anxiety, games, puzzles, and explorations are clearly

[1] Anyone who finds this is exaggerated should make the following experiment. Ask a well-educated layman how much one US dollar is worth in terms of the Canadian dollar, when the latter is pegged at 73 US cents. If he knows "the answer", ask him to defend it.

indispensible. However, a steady diet of them would not only lead to indigestion, but also be perceived as unreal and condescending. People need to feel that something significant is being developed, and that they are beginning to master it.

Good cases can be made for various exotic topics (groups, graphs, etc.) as being natural entry-points for neophytes, since they can be started from scratch and quickly lead to interesting problems. However, Math335 has opted for an introduction to the classical school curriculum. After all, prospective teachers might as well deal with questions likely to arise in their future environment: about high school math[2], about standard calculator functions, etc. Moreover, the traditional curriculum has a certain historical[3] and practical[4] legitimacy. Finally, the departmental committee at UBC had prescribed that the course should have three components: geometry, arithmetic, and combinatorics (with probability).

To ensure the theoretical cohesion of these three components, each of them was organized around a couple of simple themes, which would serve as anchors while still allowing a lot of exploratory movement around them. In geometry, the central themes are the Pythagorean theorem and the yoga of scaling[5] (which happen to be the main ingredients of trigonometry). Arithmetic includes an informal introduction to approximations, but has one focus in the relation between fractions and periodic decimals, the other in the computation and applications of (fractional) exponentials. The third component does the inevitable minimum on multiplicative counting and then settles around binomial probabilities and (by extrapolation) the bell curve.

Notation. Modern mathematical notation is a double-edged tool: for some (fortunately including most engineers) it provides a fast track into the heart of the Newtonian world, but for the vast majority it looms as a frightening obstacle. Its slick syntax can effortlessly handle subtleties (say, about negatives) which formerly befuddled great minds. However, by shifting the onus of thought toward formal operations, it has created a two-fold trap for the learner: the temptation to skip conceptualization and, at the other extreme, the futile pursuit of elusive meanings[6]. Not only mental sluggards thus fall by the wayside, but also those who were simply too critical or inquisitive at the wrong time.

For the lay public, however well-educated, the medium has become the message: math and its notation are one — formulas forever. In academic debates, this may be an acceptable epistemological stance (cf. Hilbert's Programme), but it is certainly not a valid psychological or historical one, as it would have to deny the existence of any real mathematical activity prior to the seventeenth century. The first book making systematic use of formulas was published by John Wallis in 1685. By then, the entire content of the high school curriculum was well established — most of it *very* old, with only probability (by accident) relatively new.

Ideally it should therefore be possible to give a course for prospective teachers without ever writing a formula. Math335 does not practice such total abstinence, but it does try hard to avoid "black boxes" of any kind. Thus, calculator functions (even sin and log) are not legal until their results were repeatedly approximated by hand; formulas are admitted only if they are transparent abbreviations for familiar procedures; laws and rules are banned altogether, or rather, left implicit. For instance, no "law of cosines" is invoked to compute a length BC from

[2] On her first job, one former Math335 student wound up teaching math at a senior high school.

[3] Except for its unfortunate downgrading of geometry, it gives a fair overview of pre-Cartesian math.

[4] Apart from balancing their cheque-books, most users will wish to do unit conversions, interest calculations, and simple exercises in indirect measurement (lengths, areas, and volumes).

[5] Scaling was not part of the original plan but seems to be almost everyone's Achilles heel.

[6] What exactly *is* one and a half "divided by" three point seven six? Explain by example!

an angle BAC and the lengths of its legs. Instead, an auxiliary triangle BAD is introduced, with the same angle at A and a right angle at D; its side-lengths DA and DB can be inferred from the tabulated dimensions of right angle scale models with unit hypotenuse[7]; the rest is Pythagoras. In other words, the computation re-enacts the derivation of the hidden "law", with little extra effort but a great gain in conceptual control.

Implementation. Math335 meets five hours a week. According to the UBC calendar, two of these make up a "required tutorial", but experience shows that this distinction cannot be upheld too rigidly. What happens in reality is that, once or twice a week, there is a kind of lab hour: in groups of four or five, students are engaged in some collaborative project. For instance, they might be approximating the area of the unit circle by that of a regular polygon with $6, 12, 24, \ldots, 6144$ sides. Each student acts like a line of computer code; accepting input from the neighbour on the left, processing it, and handing the output to the neighbour on the right, they go round and round the loop[8]. At the 6144-gon, their area is $3.141592\ldots$.

Not every activity is so tightly organized. Sometimes they just jointly work their way through a problem set — which usually includes a couple of items on the frontier of their current understanding. In any case, the scheduling must be kept flexible: it often happens that an unforeseen burning question postpones a lab hour to the next day.

The team structure of the class is maintained throughout the course. In almost every hour, lab or not, there comes a point where a few minutes of joint work on a problem or conceptual difficulty are required to set things straight. The amazing clarifying power of even the most muddled discussion, so familiar to professionals, also works for these students — and incidentally gives them valuable first hand experience about teaching and learning.

Of course, all that group work must be balanced by the appropriate amount of individual concentration, i.e., homework. There is only one massive assignment: to keep a detailed chronicle of their explorations, insights, and difficulties — as articulate, complete, and original as possible — written in the manner of a personal mathematical diary for serious future reference. Such a "journal" may seem strange in a math course, but it would be difficult to devise another, equally challenging, task for a class of about 40 mathemaphobia victims with diverse abilities and backgrounds[9]. It is important for them to know that a perfunctory job, too obviously produced just "for school", will not win approval. The best journal I have ever seen was handed in by a woman in her thirties, with no prior penchant for math, who had spent about 20 hours a week on it. She also said:" I feel that this is what I came back to school for."

The final exam, too, has an expository component. It contains no mere exercises, but its "problems" are neither too novel nor too difficult. There is a fair amount of choice, but students are asked to attempt at least one question from each of the three parts of the course.

Postscript. Since January 1989, Math335 has been offered in three to five classes a year. Over that time, the format described here has remained relatively stable (except for a few instances which involved deliberately different approaches). However, many subtle changes have occurred in the details, as we teachers learned more about our students. The present report was first written for the UBC math department, but was eventually thought to be of potential interest to a larger public.

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[7] These are, of course, (home-made) trig tables, but that terminology is avoided at first, in order to prevent panic and mechanical responses.

[8] This uses only Pythagoras. As a by-product, they obtain the trig tables mentioned above.

[9] Those who believe themselves saddled with a "math block" (almost always imaginary) can be encouraged to concentrate on obtaining and recording first-hand insight into learning disabilities.