

## A LOOK AT THE CONFERENCE LOGO.

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On Friday night as I rode on the bus with my new CMESG-GCDEM Conference bag, a woman sitting across the aisle from me was staring at the logo as if trying to decipher it. Feeling uneasy about the undeserved attention — I could not have told her anything intelligible about it — I decided to make amends and prepare an *ad hoc* presentation for the following day: the usual story about Fibonacci and the Golden Section, about pentagons and spirals, but — in honour of the unknown woman — using neither *algebra* nor *ratios*.

Perhaps she was an engineer or an actuary, but if her mathematical career was closer to average, she was likely left befuddled by fractions, percentages, rates, and ratios — especially irrational ones — and if she did clear that daunting set of hurdles, chances are that she finally gave up when she ran into algebra. So let us shift into a perspective she might be able to share.

Given a rectangle  $R$ , let us make a bigger one  $R'$  — and call it the *augmentation* of  $R$  — by attaching a square to the longer side of  $R$ . Further let us say that  $R$  is *golden* if  $R$  and  $R'$  are *similar* (i.e., can be lined up with sides and diagonals pairwise parallel). Easy exercises: golden rectangles are readily constructed by compass and straight-edge; if  $R$  is golden, so is  $R'$ ; a golden rectangle cannot be tiled by squares (which means its sides are *incommensurable*).

The deviation from goldenness of an arbitrary rectangle  $R$  can be measured by a snippet of area we might call the “defect” of  $R$ . Fibonacci’s rabbits are now outflanked by the following theorem: *any rectangle  $R$  and its augmentation  $R'$  have the same defect*. As repeated augmentation creates larger and larger rectangles, this defect eventually becomes relatively tiny — in other words: successive rectangles become more and more golden. And if you start with a unit square, their sides will measure 1, 2, 3, 5, 8, 13, 21, . . .

Our small *ad hoc* group dwelt so long on these details that not much was said about pentagons and spirals — except that the former are made from golden triangles (rectangles with one side collapsed), and the latter cannot be made from circular arcs (appearance notwithstanding) if they are to emulate nautilus shells.

After this seminar, I finally took a good look at my bag and made an embarrassing discovery: the diagram on the logo was not golden at all — but “bronze” at best, with each “augmentation” attaching a half-square instead of a full one, hence Fibonacci’s sequence replaced by  $1, \frac{3}{2}, \frac{7}{4}, \frac{19}{8}, \frac{47}{16}, \frac{123}{32}, \frac{311}{64}, \dots$  with quotients converging to  $1/4$  plus the square root of  $17/16$ . Maybe that is what the lady on the bus was contemplating!