Worksheet 8: Proof or disproof?

Decide whether the following statements are True or False, and prove/disprove them:

1. Let $\{u_n\}$ be a sequence of real numbers. Reminder: we say that $u_n \to +\infty$ if $\forall N > 0, \ \exists m \ (n \geq m \Rightarrow u_n > N)$.

   (a) If $\{u_n\} \to \infty$, then $-2u_n \to -\infty$.

   (b) If $u_n \to \infty$ then there does not exist m such that $u_m < 0$.

   (c) If $u_n \to \infty$ then $\forall n \ u_{n+1} \geq u_n$.

   (d) The converse of (c). (First state this converse).

   (e) If $u_n \to \infty$ then $\exists n \ u_n > 1000$.

2. $\exists x, \ \forall y \ x^2 - y < 0$.

3. $\forall y, \ \exists x \ x^2 - y < 0$.