

Worksheet 7: Quantifiers, Negation - Part 2.

1. Negate the statement:

$$\forall \epsilon > 0 \exists \delta > 0, \text{ s.t. } \forall x (|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon).$$

2. Negate the statement:

$$\forall N > 0 \exists M > 0, \text{ s.t. } \forall x (x > M \Rightarrow f(x) > N).$$

3. Write the statement ' $\exists y_0 \in \mathbb{R}, \forall x \in \mathbb{R} x^2 - 2x + 3 \geq y_0$ ' in words. Is this statement true or false? Write its negation both in symbols and in words. If this y_0 exists, is it *unique*?

4. Let $f(x)$ be some function (from the real numbers to the real numbers). Do the statements:

$$\exists y \forall x f(x) \leq f(y)$$

and

$$\forall x \exists y f(x) \leq f(y)$$

mean the same thing? Explain in words what each of them means. For each of the statements, make an example of a function that makes it true, and an example that makes it false.