Worksheet 6: Quantifiers, Negation. **The previous worksheet:** Let us make some notation: let S be the set of all students in the class, and for every student $s \in S$, denote by $F(s)$ the set of all friends of s. For a person p, let $a(p)$ be the age of p.

1. Using this notation, write in symbols the statement: "There exists a student in the class all of *whose* friends are older than him/her".

2. Make similar notation and then write symbolically the statement "There exists a tree in Stanley Park *such that* all the neighbouring trees are at least as tall as this tree."

3. Is this statement about Stanley Park true or false?

New problems

1. Negate the statement in (1).

2. Write the statement '$\exists y_0 \in \mathbb{R}, \forall x \in \mathbb{R}\ x^2 - 2x + 3 \geq y_0$' in words. Is this statement true or false? Write its negation both in symbols and in words.

3. Write in words, then negate: Let P be the set of all primes.

$$\forall N > 0, \exists p \in P, s.t.\ (p > N) \wedge (p + 2 \in P).$$

4. Negate the statement:

$$\forall \epsilon > 0 \exists \delta > 0, s.t.\ \forall x (|x - a| < \delta \Rightarrow |f(x) - f(a)| < \epsilon).$$

(Note: this statement is the definition of '$f(x)$ is continuous at a') (think of it!)

5. Negate the statement:

$$\forall N > 0 \exists M > 0, s.t.\ \forall x (x > M \Rightarrow f(x) > N).$$

(This is the definition of $\lim_{x \to +\infty} f(x) = +\infty$).