Worksheet 1: Sets and Logic

1. Which of the following sentences are *statements* in the mathematical sense? For the ones that are statements, can you decide whether they are True or False?

   (a) It is sunny outside right now.

   (b) Tomorrow the weather will be nice.

   (c) The 100th digit of the decimal expansion of π is 7.

   (d) The digits of π encode the meaning of the Universe.

   (e) This statement is False.

   (f) This statement is True.

   (g) For any consistent system of axioms, there exists a statement about natural numbers that is true, but unprovable from these axioms.

   (h) For some prime numbers p, the number $p + 2$ is also prime.

   (i) For all prime numbers p, the number $p + 2$ is also prime.

   (j) There exist infinitely many primes p such that the number $p + 2$ is also prime.

2. Are the following sets empty or not? When not empty, draw the set.

   (a) The set of all $x \in \mathbb{R}$ such that $x^2 > 4$ and $x < 0$.

   (b) The set of all $x \in \mathbb{R}$ such that $x^2 > 4$ and $|x| < 2$.

   (c) The set of all $x \in \mathbb{R}$ such that $x^2 \geq 4$ and $|x| \leq 2$.

   (d) The set of all $(x, y) \in \mathbb{R}^2$ such that $x^2 + y^2 = 1$ and $x < 0$.

   (e) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = -1\}$.

   (f) $\{(x, y) \in \mathbb{R}^2 : x^2 - y^2 = -1\}$.