Worksheet 19: Cardinality 2: Countable sets (and a bit of review)

Definition. The sets A is called *countably infinite* if $|A| = |\mathbb{N}|$.

1. Prove that the set of even numbers has the same cardinality as $\mathbb{N}$.
2. Prove that $|\mathbb{Z}| = |\mathbb{N}|$.
3. Prove that $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$.
4. Prove that $|\mathbb{Q}| = |\mathbb{N}|$.
5. Prove directly that $[0, 1)$ and $(0, 1)$ have the same cardinality.

Once we have done this, we will go through Cantor's proof (in the book, p. 271) that $|\mathbb{N}| \neq |\mathbb{R}|$.