Worksheet 18: Cardinality: the same or not the same?

Definition. The sets A and B have the same cardinality if there exists a bijective function $f : A \to B$. This is denoted by |A| = |B|.

1. Prove that any two intervals (a, b) and (c, d) on the real line have the same cardinality.

2. Prove that the interval (0, 1) has the same cardinality as the set of real numbers \mathbb{R} .

3. Prove that the set of even numbers has the same cardinality as \mathbb{N} .

1

4. Prove that $|\mathbb{Z}| = |\mathbb{N}|$.

Once we have done this, we will go through Cantor's proof (in the book, p. 271) that $|\mathbb{N}| \neq |\mathbb{R}|$.