Worksheet 16: Inverse functions. Images and preimages of sets.

Recall the definition (from 12.6): if $f : A \to B$ is a *bijective* function, then there is a well-defined *inverse function* $f^{-1} : B \to A$.

1. (from the last worksheet). Prove that if $f : A \to B$ is injective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is injective. Is the converse statement true?

2. Does the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \tan(x)$ have an inverse? How can we modify this function to make the inverse function well-defined?

3. True or false: $\arcsin(\sin(x)) = x$ for all $x \in \mathbb{R}$? What about $\sin(\arcsin(x)) = x$ for all $x \in [-1, 1]$? Write both statements in terms of composition of functions.

Prove every statement in Theorem 12.4 on p. 243 (see also the posted notes).