Worksheet 15: Review functions: injective, surjective, bijective functions. Range.

(solutions will be posted on Wednesday)

1. Determine the range of the functions $f : \mathbb{R} \to \mathbb{R}$ defined as follows:

   (a) $f(x) = \dfrac{x^2}{1+x^2}$

   (b) $f(x) = \dfrac{x}{1+|x|}$

2. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be defined by

   $$f(a,b) = \frac{(a+1)(a+2b)}{2}$$

   Show that the image of f is contained in $\mathbb{N}$, so that $f : \mathbb{R} \to \mathbb{N}$ is a well-defined function.

3. Explain why multiplication by 2 defines a bijection from $\mathbb{R}$ to $\mathbb{R}$, but not from $\mathbb{Z}$ to $\mathbb{Z}$.

4. Write four different bijections $f : \mathbb{N} \to \mathbb{N}$.

5. *Final Exam - Dec 2010* Prove that the following function is bijective

   $$f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{1\} \text{ defined by } f(x) = \frac{x+1}{x+2}$$