Worksheet 14: Injective and surjective functions; composition.

1. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is injective but not surjective. Can you make such a function from a finite set to itself?

2. Prove that the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined by $f(a, b) = 3a + 7b$ is surjective. Is this function injective?

3. Prove that among any six distinct integers, there are two whose difference is divisible by 5.

4. Let $f : \{1, 2, 3\} \to \{a, b, c, d\}$ be defined by $f(1) = a$, $f(2) = c$, $f(3) = d$. Let $g : \{a, b, c, d\} \to \{1, 2, 3, 4, 5\}$ be defined by $g(a) = 2$, $g(b) = 1$, $g(c) = 4$, $g(d) = 5$. Find the composition $g \circ f$.

5. Prove that if $f : A \to B$ is injective and $g : B \to C$ is injective, then $g \circ f : A \to C$ is injective. Is the converse statement true?

6. Prove that among any ten points located on a circle with diameter 5, there exist at least two at a distance less than 2 from each other.

7. Show that in any group of n people, there are two who have the same number of friends within the group.