

Worksheet 11: Sets; indexed collections

1. Let A, B, C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

2. Let $T_a = \{x \in \mathbb{R} : x \geq 0 \wedge x < a - 2\}$. Prove that if $T_a = \emptyset$ then $a \leq 2$.

What is wrong with the following start of an argument:

“Since $x \geq 0$ and $x < a - 2$ we must have $0 \leq x < a - 2$.
Then ... ”

3. Let $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ be any function. For $N \in \mathbb{N}$, let $A_N = \{x \in \mathbb{R} : f(x) > N\}$.

Prove that:

- (a) $\lim_{x \rightarrow +\infty} f(x) = +\infty$ if and only if for every N , there exists $m > 0$ such that $A_N \supseteq (m, +\infty)$.
- (b) Prove that $\bigcap_{N \in \mathbb{N}} A_N = \emptyset$.