Problem 3 on worksheet from last class: Thurs. Jan 16

Lemma: (from last class)

Let n be an integer.

If $2 | n$ and $3 | n$, then $6 | n$.

Proof: Since $3 | n$, by definition of divisibility, we have $n = 3k$ for some $k \in \mathbb{Z}$.

If we show that k is even, we'll be done.

There are two possibilities: k is even ← we like it

or k is odd.

proof by contradiction: assume a possibility we want to rule out, arrive at a contradiction

We want to rule out the possibility k odd.

If k were odd, then we would have $k = 2\ell + 1$ for some $\ell \in \mathbb{Z}$.

Then (plug it into the expression for n):

$$n = 3(2\ell+1) = 6\ell + 3 = 2\underbrace{(3\ell+1)}_{\substack{\in \mathbb{Z} \\ \text{b/c } \ell \in \mathbb{Z}}} + 1 \text{ - odd,}$$

which contradicts the assumption that n is even.

So k cannot be odd (otherwise n would be odd, which contradicts our assumption)

Then k is even,

so $k = 2m$ for ~~some~~ some $m \in \mathbb{Z}$.

Put it together: $n = 3k = 3 \cdot (2m) = 6m$,

so n is divisible by 6. ▪

From Lemma to our problem:

Let $a = n(n+1)(n+2)$

We want to prove: a is even and $3 | a$.

Then by Lemma, we conclude $6 | a$.

a is a product of 3 consequitive integers;
at least one of them has to be even,
and at least ~~a~~ one of them has to
be divisible by 3.

Then the product is even, and ~~divisible~~ divisible by 3.

(Homework: if a is even, $b \in \mathbb{Z}$ $\Rightarrow$ $a \cdot b$ is even
if a is div. by 3, $b \in \mathbb{Z}$ $\Rightarrow$ $a \cdot b$ is divisible by 3).

Different proof of the main statement:

- Accept (will prove later): Division algorithm:

~~[crossed out]~~, $a \in \mathbb{Z}$, $b \in \mathbb{N}$

Then there are unique $q \in \mathbb{Z}$ and $r \in \mathbb{Z}$ such that

$a = bq + r$ and $0 \le r < b$

(can divide a by b with remainder)

unique: there is only one pair (q, r) for given a, b.

Can do proof by cases:

possible remainders when dividing n by 6 are:

$0, 1, 2, 3, 4, 5$.

The remainder of $n(n+1)(n+2)$ when dividing by 6 "modulo"
depends only on the remainder of n modulo 6. mod 6.

interesting statement. Will prove it in a month or so.

See next page

The blue statement says:

if n_1, n_2 have the same remainder mod 6
then
$n_1(n_1+1)(n_1+2)$ and $n_2(n_2+1)(n_2+2)$
also have the same remainder mod 6.

Believe this for now.

Then <u>prove by cases</u>:

- if n is 0 mod 6
  then $\underline{\underline{n}}(n+1)(n+2)$ is divisible by 6.
- if n has remainder 1 mod 6
  then $n(n+1)(n+2)$ has remainder same as $\to$ $1\cdot(1+1)(1+2)$
  $= 1\cdot 2\cdot 3 = 6$
  – divible by 6
- if n has remainder 2 mod 6.
  $n(n+1)(n+2)$ has the same remainder as
  $2\cdot(2+1)(2+2) = 2\cdot 3\cdot 4$, which is divisible by 6.

trick question:
can I use WLOG
"without loss of generality"
here to conclude the proof?

NO! Here different remainders could give a different answer – we do not know till we check!

Have to do cases n has remainder 3, 4, 5 mod 6
but I will skip them. ← in homework if doing cases, have to do all.

Done with Chap. 4

Next: 2.4, 2.5, 2.6.

~~[illegible]~~ Last piece of logic was $P \Rightarrow Q$.

- Converse : $Q \Rightarrow P$ — today
- contrapositive : $\sim Q \Rightarrow \sim P$
- negation : $\sim (P \Rightarrow Q)$

our main topic ~~today~~ next class.

First we need to define "logical equivalence"

Biconditionals and logical equivalence

- biconditional statement is : $P \Leftrightarrow Q$

(LaTeX : \Leftrightarrow)

"P if and only if Q"

By definition : $P \Leftrightarrow Q$ is a new statement with the truth table:

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

the same as $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

Worksheet 3: Converse, biconditional, truth tables

1. Break the following statement into simple statements (they are not allowed to include the word 'not'), express it in logical symbols, and construct the truth table for it.

   'I will go to UBC this week if it is not a snow day and the date is even'.

2. Does the above statement imply that I will not be at UBC on Friday January 17? NO! ← assumption is false for Friday, so anything can happen.

3. Formulate the converse to the above statement.

4. Make a true biconditional statement about my schedule this week.

I will go to UBC if and only if it is (Tue, Th or Fri) and not a snow day

1) P : I go to UBC

Q : it is a snow day

R : date is even

$$\boxed{((\sim Q) \wedge R) \Rightarrow P}$$

$\Leftarrow$

$\updownarrow$

"If it is not a snow day and the date is even, I will come to UBC"

P if Q and Q if P mean the same thing.

(Note: colloquially, we often mean the other implication (i.e. the converse), when we say it this way)

↑

NOT a math interpretation.

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3) The converse:

$$P \Rightarrow (\sim Q \wedge R)$$

if you see me at UBC then it's not a snow day and the date is even.

Another way to state it:

I will go to UBC only if it's not a snow day and the date is even

Truth tables:

$\sim Q \wedge R \Rightarrow P$ ← statement composed of P, Q, R

input | output

P	Q	R	$\sim Q \wedge R \Rightarrow P$	$\sim Q$	$\sim Q \wedge R$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	T	T	F

2^3 possibilities

fill out first | then this

Logical equivalence: Two statements are logically equivalent if they are T or F at the same time (have same Truth tables).

P and Q logically equivalent is the same as ($P \Leftrightarrow Q$ is True)

We reserve "logically equivalent" just for composite logical statements; others are just biconditionals

EX: 1) n is even $\Leftrightarrow$ n^2 is even is a True biconditional
I do not call it logical equivalence

2) $(P \Leftrightarrow Q)$ is logically equivalent to $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

– it is a statement about statements.

so it is a logical equivalence.