- <u>Induction</u>: HW 5 – not to turn in.

<u>Recommend</u>: write it and exchange with a friend, mark each other's.

Turn it in after the break, +2 to total HW.
+1 w/o group <u>marking</u>.

HW 6: Feb 28?
<u>27</u> ← more likely.

- <u>Sets</u> sections 1.2 – 1.8 (Tuesday)

<u>Def</u>: A, B – sets.

1) We say <u>$B \subset A$</u> "B <u>is a subset</u> of A" – <u>statement</u>. (True or false for given two sets).

\subset

if every element of B is also an element of A:

$(\forall x \in B,\ x \in A)$ or, $(x \in B \Rightarrow x \in A)$

2) $A \cap B$ – the <u>intersection</u> of A and B

\cap

$A \cup B$ – the <u>union</u> of A and B

TeX \cup

→ $A \cap B = \{ x \in A \mid x \in B \}$ – the set of common elements of A and B

new set.

(if A, B have no common elements, then $A \cap B = \emptyset$)

also a new set – $A \cup B$ = the set of all elements of A, and all elements of B

$(x \in A \cup B \iff x \in A \text{ or } x \in B.)$

Indexed collections of sets

suppose for every natural number n, you are given a set (call it A_n):

example: $A_{n} = \{ m \in \mathbb{Z} : n \mid m \}$

Then $A_1 = \mathbb{Z}$

A_2 = even integers

A_3 = integers divisible by 3, — —

...

$\boxed{n \text{ is the index}}$

This is an example of 'indexed collection of sets'.

[We had one on the exam:

$T_{a} = \{ x \in \mathbb{R} : x \geq 0 \wedge x < a-2 \}$

collection of sets indexed by $\mathbb{R}$

$\boxed{a \text{ is the index}}$

your set depends on a parameter from some set S (in our examples, $S = \mathbb{N}$, $S = \mathbb{R}$)

then you have a collection of sets indexed by S.

Can make: $A := \bigcup_{s \in S} A_s$ — union of all of them

or $B := \bigcap_{s \in S} A_s$ - intersection of all of them

— More on Tuesday. — —

Strong induction example

- To celebrate the 7^{th} year of dictatorship, Trump issues new notes: \$3 and \$7. (prohibits all else).

Prove that you can pay any amount $\geq \$12$ by using only these notes, and at most two \$7.

Mathematically: $\forall\, a \geq 12,\ \exists\, n, m \in \mathbb{N} \cup \{0\}$ such that $a = 3n + 7m$. and $m \leq 2$

Proof: Try $a = 12$:

$12 = 4 \cdot 3$ ok

$13 = 7 + 2 \cdot 3$ ok

$14 = 2 \cdot 7$ ok

not needed → $15 = 3 \cdot 5$ - ok.

Why did we have to check so many base cases?

see below.

Induction: Suppose that for all amounts $12 \leq b \leq a$ we can pay.

Want to prove: can pay $a+1$.

Consider the number $(a+1) - 3 = a - 2$

By the induction assumption, we have some m, n:

$$a - 2 = 3n + 7m \quad \text{and} \quad m \leq 2$$

Then $a + 1 = 3(n+1) + 7m$, and we are done.

For this argument to work, we need the induction assumption to be true for $a-2$.
This means, we need $a - 2 \geq 12$, so $a \geq 14$.
Thus, induction step works for passing from 14 to 15, but it does NOT work for passing from 13 to 14.
Because of this, $a = 12$, 13, and 14 all need to be checked as base cases. Note: this is exactly where the error was hiding in #3 in the worksheet 9 from last class (where you needed to find the error).