1. (a) Prove that there are infinitely many primes p such that $p \equiv 3 \mod 4$. *Hint: try to proceed the same way as in Euclid's proof of the statement that there are infinitely many prime numbers; but instead of making the number* $N = p_1, \dots p_n + 1$, *make a number* N *that is definitely congruent to* 3 *modulo* 4 *(and that still differs by* 1 *from a number that is divisible by all of* $p_1, \dots p_k$*).*

   (b) Could this proof have worked for the primes congruent to 1 modulo 4?

2. Prove that the number 123456782 cannot be represented as $a^2 + 3b^2$ for any integers a and b. (*Hint: Consider the remainder* $\mod 3$).

3. Prove that there do not exist integers a, b and c such that

$$12345678910111213 = a^2 + 25b^2 + 5c^2.$$