Math 263 Midterm I (October 3, 2007)
Section 102. Instructor: Julia Gordon

Problem 1:

Consider two intersecting lines

$$L_1 : \quad x - 4 = -\frac{1}{2}y = \frac{1}{5}(z - 7)$$
$$L_2 : \quad \frac{1}{2}x + 2 = \frac{1}{3}(y + 5) = -z$$

(a) [*4 points*] Find the intersection of L_1 and L_2.

(b) [*4 points*] Find the acute angle between L_1 and L_2.

(c) [*5 points*] Find the equation of the plane containing L_1 and L_2.

Problem 2: One particle is moving along the straight line $\mathbf{r}_1(t) = \langle 2\pi + t, 2t, 1 + t \rangle$, and another one is moving along the helix $\mathbf{r}_2(t) = \langle t, \sin(t), \cos(t) \rangle$.

(a) [*4 points*] Would the particles collide?

(b) [*4 points*] Do their trajectories intersect?

(c) [*5 points*] Find the tangential and normal components of the acceleration of the second particle when it is at the point $(2\pi, 0, 1)$.

Problem 3:

Consider the function $z(x, y) = 4 - x^2 - y^2$.

(a) [*4 points*] Sketch the surface represented by $z(x, y)$. Determine if $z(x, y)$ is continuous at $(x_0, y_0) = (1, 0)$.

(b) [*5 points*] Compute z, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(x_0, y_0) = (1, 0)$.

(c) [*5 points*] Use linear approximation to estimate $z(1, 0.1)$, and compare your approximation with the exact value.