

Table of integrals to remember. The $+C$ is omitted everywhere.

$$\int x^\alpha dx = \frac{1}{\alpha + 1} x^{\alpha+1} \quad \text{for all real numbers } \alpha \neq -1.$$

$$\int x^{-1} dx = \ln |x|.$$

$$\int e^x dx = e^x.$$

$$\int a^x dx = \frac{1}{\ln a} a^x.$$

$$\int \ln(x) dx = x \ln x - x.$$

$$\int \sin(x) dx = -\cos(x);$$

$$\int \cos(x) dx = \sin(x).$$

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x)$$

$$\int \frac{1}{1+x^2} = \tan^{-1}(x)$$

$$\int \frac{1}{x^2 - a^2} = \frac{1}{2a} (\ln |x - a| - \ln |x + a|) = \frac{1}{2a} \ln \left(\frac{|x - a|}{|x + a|} \right).$$

Note: if in the trigonometric integrals above, instead of $1 - x^2$ or $1 + x^2$ you have $a^2 - x^2$ or $a^2 + x^2$, then use the substitution $u = x/a$; if you have some other quadratic polynomial, then *complete the square*, and reduce it to $a^2 - x^2$, $a^2 + x^2$, or $x^2 - a^2$.

Also, remember the following basic techniques:

- substitution.
- For expressions such as $x^n \ln x$ or $x^n e^x$, or $x^n \sin(x)$, $x^n \cos(x)$, use integration by parts.
- The strategy for integrating $\sin^n(x) \cos^m(x) dx$ is to make a substitution $u = \cos(x)$ or $u = \sin(x)$ (use the function whose power is odd); if both powers are even, use double-angle formulas to reduce to smaller powers of the functions of $2x$.

The main double angle formula:

$$\cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1.$$

$$\text{Then } \cos^2 x = \frac{\cos(2x)+1}{2}.$$