Background for Lab 1

This laboratory is concerned with the question

> " If we drop an object from a distance 10,000 km from the centre of the Earth, how long does it take for it to fall to the surface of the Earth?"

In order to answer this question, we need to start with a few facts:

- The surface of the Earth is $R = 6,370\ km$ from the centre.
- The acceleration of gravity at the Earth's surface is 9.8 m/sec^2.
- The force of gravity is proportional to an object's mass and inversely proportional to the square of the distance from the Earth's centre.

Hence the force on an object at radius r is $-\frac{mg}{(r/R)^2}$. The sign is negative because the force acts downward. Newton's Second Law, $F = ma$, therefore gives us the **differential equation**

$$r'' = -\frac{g}{(r/R)^2} = -\frac{gR^2}{r^2} \tag{I}$$

Note that the sign is negative. If you drop an object, it falls. So r decreases with time.

We do not know how to solve this equation explicitly. In practice, it is usually sufficient to solve equations approximately and that is what we shall do in this lab. We shall find approximate values for the position and velocity at times 0, dt, $2dt$, ... separated by intervals of length dt. The approximate position, velocity and acceleration at time ndt are denoted r_n, v_n and a_n respectively. We shall assume that the position and velocity at time zero are

$$r_0 = 10000 \qquad v_0 = 0$$

The differential equation (I) then tells us that the acceleration at time 0 is

$$a_0 = -\frac{gR^2}{r_0^2} = -\frac{9.8 \times 6370^2}{10000^2}$$

If the velocity were to remain constant at v_0 throughout the entire time interval from $t = 0$ to $t = dt$, then, because velocity is rate of change of radius, the radius at time dt would be the radius at time zero, r_0, plus the amount, $v_0 \times dt$, that the radius would change over the time interval. So, the radius at time dt would be $r_1 = r_0 + v_0 \times dt = 10000$. We take this as our approximate value for $r(dt)$. Similarly, if the acceleration were to remain constant at a_0 throughout the entire time interval from $t = 0$ to $t = dt$, then, because acceleration is rate of change of velocity, the velocity at time dt would be $v_1 = v_0 + a_0 \times dt$. We take this as our approximate value for $v(dt)$. We now have approximate values for the position and velocity at time dt, namely r_1 and v_1. By the differential equation (I), the acceleration at position r_1 is $a_1 = -\frac{gR^2}{r_1^2}$. We take this as the approximate value for the acceleration at time dt.

If the velocity were to remain constant at v_1 throughout the entire time interval from $t = dt$ to $t = 2dt$, then, the radius at time $2dt$ would be $r_2 = r_1 + v_1 \times dt$. We take this as our approximate value for $r(2dt)$. Similarly, if the acceleration were to remain constant at a_1 throughout the entire time interval from $t = dt$ to $t = 2dt$, then, the velocity at time $2dt$ would be $v_2 = v_1 + a_1 \times dt$. We take this as our approximate value for $v(2dt)$. By the differential equation (I), the acceleration at position r_2 is $a_2 = -\frac{gR^2}{r_2^2}$. We take this as the approximate value for the acceleration at time $2dt$.

Lab 1 has a built in computer programme that repeats this approximation procedure over and over many times. Your job in the lab will be to try and determine experimentally how the accuracy of this approximation procedure depends on the size dt of the time interval.