

Substitutions for Integrating Trigonometric Functions

y	dy		good for
$\sin x$	$\cos x dx$	$\sin x = y, \cos^2 x = 1 - y^2$	$\int \sin^m x \cos^n x dx$ with $n \geq 1$, odd
$\cos x$	$-\sin x dx$	$\sin^2 x = 1 - y^2, \cos x = y$	$\int \sin^m x \cos^n x dx$ with $m \geq 1$, odd
$\tan x$	$\sec^2 x dx$	$\tan x = y, \sec^2 x = 1 + y^2$	$\int \tan^m x \sec^n x dx$ with $n \geq 2$, even
$\cot x$	$-\csc^2 x dx$	$\cot x = y, \csc^2 x = 1 + y^2$	$\int \cot^m x \csc^n x dx$ with $n \geq 2$, even
$\sec x$	$\sec x \tan x dx$	$\sec x = y, \tan^2 x = y^2 - 1$	$\int \tan^m x \sec^n x dx$ with $m \geq 1$, odd, $n \geq 1$
$\csc x$	$-\csc x \cot x dx$	$\csc x = y, \cot^2 x = y^2 - 1$	$\int \cot^m x \csc^n x dx$ with $m \geq 1$, odd, $n \geq 1$

Substitutions for Integrating Square Roots

Integral	Substitution		
$\int (a + bx^2)^{m/2} x^n dx$ with $n \geq 1$, odd	$y = a + bx^2$	$dy = 2bx dx$	$a + bx^2 = y, x^2 = \frac{1}{b}(y - a)$
$\int (a^2 - x^2)^{m/2} x^n dx$ with $n \geq 0$, even	$x = a \sin y$	$dx = a \cos y dy$	$\sqrt{a^2 - x^2} = a \cos y $
$\int (a^2 - b^2x^2)^{m/2} x^n dx$ with $n \geq 0$, even	$bx = a \sin y$	$dx = \frac{a}{b} \cos y dy$	$\sqrt{a^2 - b^2x^2} = a \cos y $
$\int (a^2 + x^2)^{m/2} x^n dx$ with $n \geq 0$, even	$x = a \tan y$	$dx = a \sec^2 y dy$	$\sqrt{a^2 + x^2} = a \sec y $
$\int (a^2 + b^2x^2)^{m/2} x^n dx$ with $n \geq 0$, even	$bx = a \tan y$	$dx = \frac{a}{b} \sec^2 y dy$	$\sqrt{a^2 + b^2x^2} = a \sec y $
$\int (x^2 - a^2)^{m/2} x^n dx$ with $n \geq 0$, even	$x = a \sec y$	$dx = a \sec y \tan y dy$	$\sqrt{x^2 - a^2} = a \tan y $
$\int (b^2x^2 - a^2)^{m/2} x^n dx$ with $n \geq 0$, even	$bx = a \sec y$	$dx = \frac{a}{b} \sec y \tan y dy$	$\sqrt{b^2x^2 - a^2} = a \tan y $