Outline

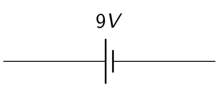
Week 5: Circuits

Course Notes: 3.5

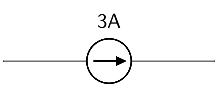
Goals: Use linear algebra to determine voltage drops and branch currents.

Notes

Components in Resistor Networks



voltage source

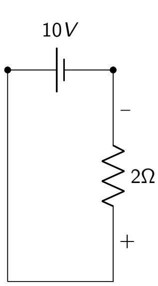


current source (inductor at an instant)

resistor

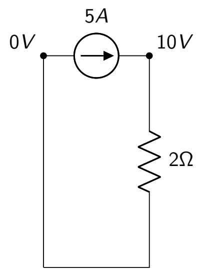
Notes

$V = IR$



Notes

$V = IR$



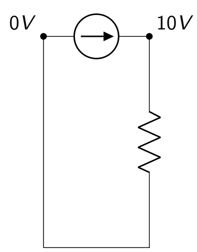
$V = 10$
(voltage drop of 10 Volts across resistor)

Setup: Given: Resistance of resistors; voltage across voltage sources; current through current sources.
Find: currents through each resistor and each voltage source; voltage drops across each current source

Notes

Kirchhoff's Laws

1. The sum of voltage drops around any closed loops in the network must be zero.
2. For any node, current in equals current out



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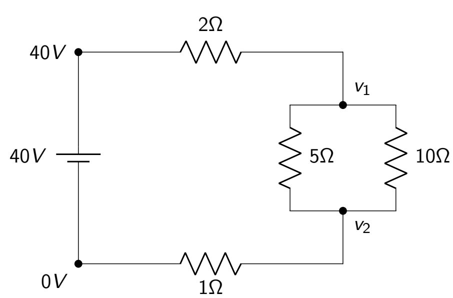
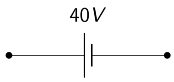


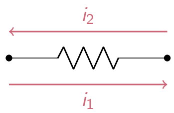
Notes

Things to Keep in Mind

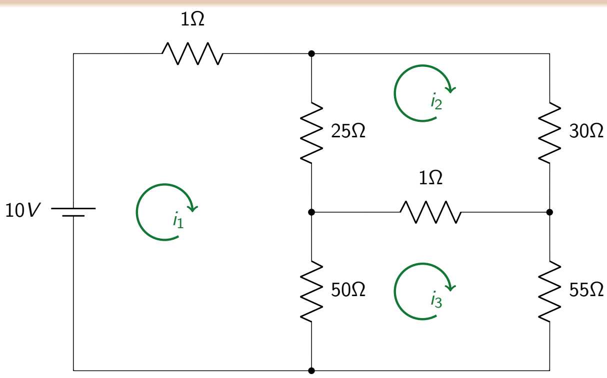
- Set up your loop currents in any direction (clockwise or counter-clockwise), then follow them around in that direction.
- If your actual flow is not in the direction you chose, you'll simply get a negative number for your current
- We're counting up voltage drops around a loop. A voltage DROP is high to low.



- Branch current is the NET effect of the loop currents.



Notes



$i_1 \approx 0.2449, \quad i_2 \approx 0.1114, \quad i_3 \approx 0.1166$

Notes

Equations from previous slide:

i_1 **loop:** $-10 + i_1 + 25(i_1 - i_2) + 50(i_1 - i_3) = 0$

i_2 **loop:** $25(i_2 - i_1) + 30i_2 + (i_2 - i_3) = 0$

i_3 **loop:** $50(i_3 - i_1) + (i_3 - i_2) + 55i_3 = 0$

$$\begin{aligned} 76i_1 - 25i_2 - 50i_3 &= 10 \\ -25i_1 + 56i_2 - i_3 &= 0 \\ -50i_1 - i_2 + 106i_3 &= 0 \end{aligned}$$

Notes

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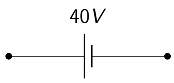

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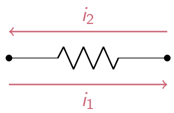

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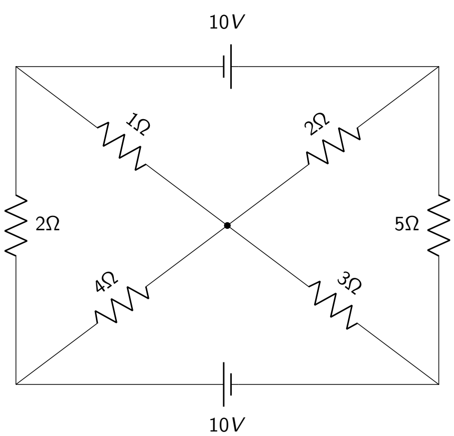


Notes

Equations from Previous Slide:

i_1 **loop:** $-10 + 2(i_1 - i_4) + (i_1 - i_2) = 0$

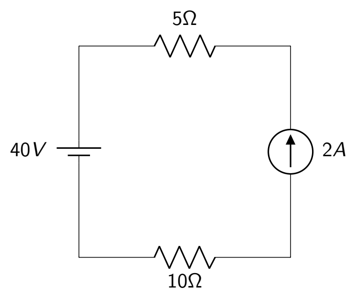
i_2 **loop:** $2i_2 + (i_2 - i_1) + 4(i_2 - i_3) = 0$

i_3 **loop:** $-10 + 4(i_3 - i_2) + 3(i_3 - i_4) = 0$

i_4 **loop:** $5i_4 + 3(i_4 - i_3) + 2(i_4 - i_1) = 0$

$$\begin{array}{rcrcrcrcr}
3i_1 & - & i_2 & + & 0i_3 & - & 2i_4 & = & 10 \\
-i_1 & + & 7i_2 & - & 4i_3 & + & 0i_4 & = & 0 \\
0i_1 & - & 4i_2 & + & 7i_3 & - & 3i_4 & = & 10 \\
-2i_1 & + & 0i_2 & - & 3i_3 & + & 10i_4 & = & 0
\end{array}$$

Notes



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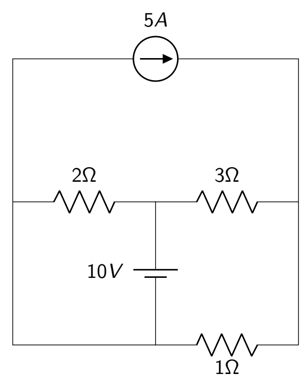

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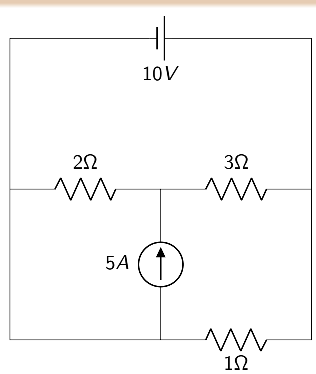


Let E be the voltage drop across the current source.

Notes

Equations from previous slide:

Current Source: $5 = i_3 - i_2$

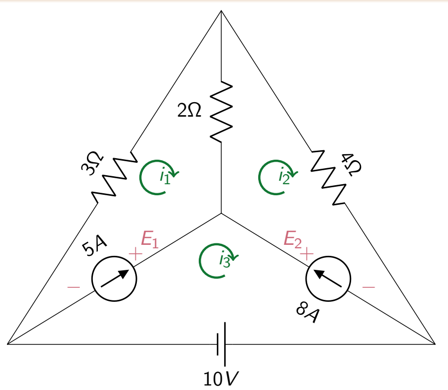
i_1 **Loop:** $-10 + 3(i_1 - i_3) + 2(i_1 - i_2) = 0$

i_2 **Loop:** $2(i_2 - i_1) + E = 0$

i_3 **Loop:** $-E + 3(i_3 - i_1) + i_3 = 0$

$$\begin{array}{rcrcrcrcr}
0i_1 & - & i_2 & + & i_3 & + & 0E & = & 5 \\
5i_1 & - & 2i_2 & - & 3i_3 & + & 0E & = & 10 \\
-2i_1 & + & 2i_2 & + & 0i_3 & + & E & = & 0 \\
-3i_1 & + & 0i_2 & + & 4i_3 & - & E & = & 0
\end{array}$$

Notes



$i_1 \approx -8.8571,\quad i_2 \approx 4.1429,\quad i_3 \approx -3.8571,$

$E_1 \approx 52.5714,\quad E_2 \approx 42.5714$

Notes

Equations from previous slide:

5A Current Source: $i_3 - i_1 = 5$

8A Current Source: $i_2 - i_3 = 8$

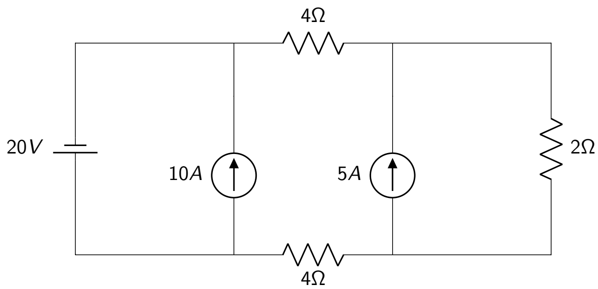
i_1 **Loop:** $3i_1 + 2(i_i - i_2) + E_1 = 0$

i_2 **Loop:** $2(i_2 - i_1) + 4i_2 - E_2 = 0$

i_3 **Loop:** $-E_1 + E_2 + 10 = 0$

$$\begin{array}{rcrcrcrcrcr}
-i_1 & + & 0i_2 & + & i_3 & + & 0E_1 & + & 0E_2 & = & 5 \\
0i_1 & + & i_2 & - & i_3 & + & 0E_1 & + & 0E_2 & = & 8 \\
5i_1 & - & 2i_2 & + & 0i_3 & + & E_1 & + & 0E_2 & = & 0 \\
-2i_1 & + & 6i_2 & + & 0i_3 & + & 0E_1 & - & E_2 & = & 0 \\
0i_1 & + & 0i_2 & + & 0i_3 & - & E_1 & + & E_2 & = & -10
\end{array}$$

Notes



Notes

Equations from previous slide:

10A Current Source: $i_2 - i_1 = 10$

5A Current Source: $i_3 - i_2 = 5$

i_1 **Loop:** $20 + E_1 = 0$

i_2 **Loop:** $4i_2 + E_2 + 4i_2 - E_1 = 0$

i_3 **Loop:** $2i_3 - E_2 = 0$

$$\begin{array}{rcrcrcrcrcr}
-i_1 & + & i_2 & + & 0i_3 & + & 0E_1 & + & 0E_2 & = & 10 \\
0i_1 & - & i_2 & + & i_3 & + & 0E_1 & + & 0E_2 & = & 5 \\
0i_1 & + & 0i_2 & + & 0i_3 & + & E_1 & + & 0E_2 & = & -20 \\
0i_1 & + & 8i_2 & + & 0i_3 & - & E_1 & + & E_2 & = & 0 \\
0i_1 & + & 0i_2 & + & 2i_3 & + & 0E_1 & - & E_2 & = & 0
\end{array}$$

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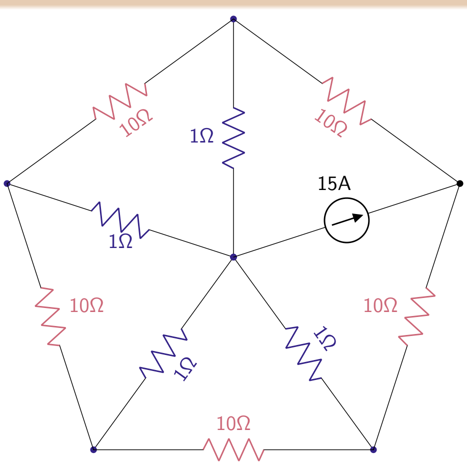

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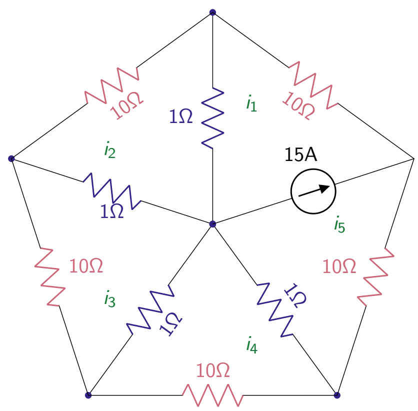

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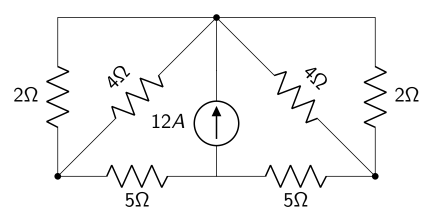
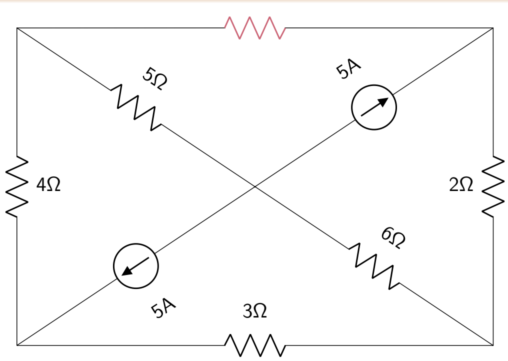


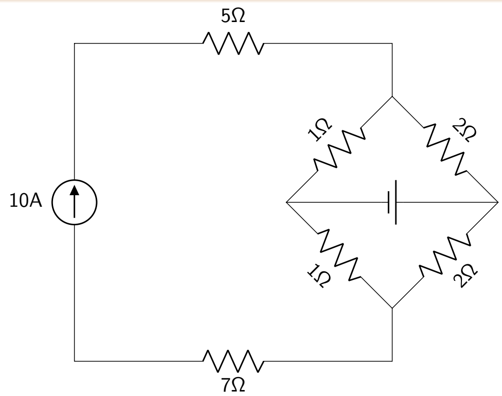
Find all branch currents.

Notes



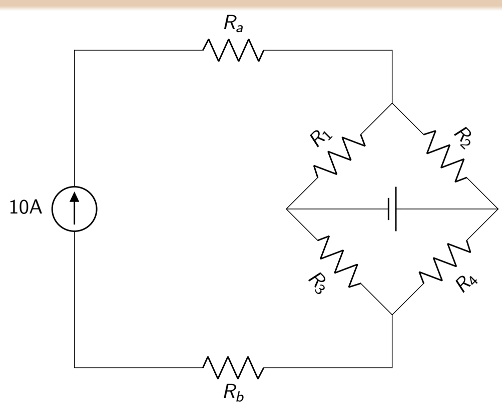
What resistance should the top resistor have, if you want each wire touching the centre to have current 5A?

Notes



What voltage should the voltage source have, in order for there to be no current across it?

Notes



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