Outline

Week 2: Determinants, Cross Products, Lines, and Planes

Course Notes: 2.4-2.5

Goals: Introduce determinants and cross products, computationally and with geometric interpretations. Lines and planes.

Matrices!

$$\begin{bmatrix} 8 & 15 & -4 \\ 9 & -4 & 7 \\ 6 & 1 & 1 \\ -5 & -3 & 0 \end{bmatrix}$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Determinants in Two and Three Dimensions

$$\det\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1b_2 - a_2b_1$$

$$\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1\det\begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2\det\begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3\det\begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

Determinants in Two and Three Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Tricky way: ONLY in three dimensions:

$$\det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

$$\det \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} -2 & 8 \\ 3 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} -2 & 8 \\ 3 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} 3 & 2 & 5 \\ 5 & 7 & 3 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} -2 & 8 \\ 3 & 5 \end{bmatrix}$$

$$\det \begin{bmatrix} 3 & 2 & 5 \\ 5 & 7 & 3 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 2 & 4 & 8 \\ 3 & 5 & 7 \\ 1 & 10 & 5 \end{bmatrix}$$

$$\det\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix} \qquad 5-6=-1$$

$$\det\begin{bmatrix}-2 & 8\\ 3 & 5\end{bmatrix} \qquad -10-24=-34$$

$$\det\begin{bmatrix}3 & 2 & 5\\ 5 & 7 & 3\\ 2 & 1 & 3\end{bmatrix} \qquad 3(21-3)-2(15-6)+5(5-14)$$

$$\det\begin{bmatrix}2 & 4 & 8\\ 3 & 5 & 7\\ 1 & 10 & 5\end{bmatrix} \qquad 2(25-70)-4(15-7)+8(30-5)$$

Geometric Interpretation: Determinant in Two Dimensions

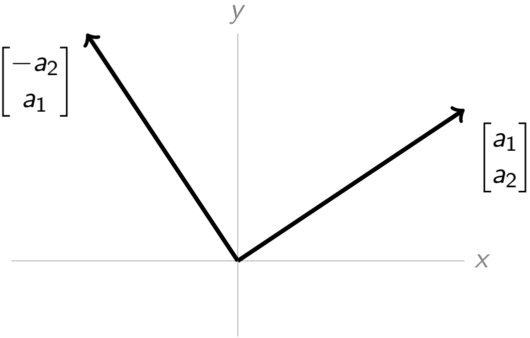
$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$



Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

If $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = 0$,

then

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

If $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = 0$,

then $\begin{bmatrix} -a_2 \\ a_1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

If $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = 0$,

then $\begin{bmatrix} -a_2 \\ a_1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

so $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ are

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

If $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = 0$,

then $\begin{bmatrix} -a_2 \\ a_1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

so $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ are scalar multiples of one another (parallel).

Geometric Interpretation: Determinant in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1$$
$$= \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

If $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = 0$,

then $\begin{bmatrix} -a_2 \\ a_1 \end{bmatrix}$ is orthogonal to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

so $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ are scalar multiples of one another (parallel).

A 2×2 matrix has determinant 0 if and only if one if its row vectors is a scalar multiple of the other.

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det\begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix}$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \neq 0$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \neq 0$$

$$\det \begin{bmatrix} a_1 & a_2 \\ 5a_1 & 5a_2 \end{bmatrix}$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \neq 0$$

$$\det \begin{bmatrix} a_1 & a_2 \\ 5a_1 & 5a_2 \end{bmatrix} = 0$$

Zero Determinants

Are the following determinants zero, or nonzero?

$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \neq 0$$

$$\det \begin{bmatrix} a_1 & a_2 \\ 5a_1 & 5a_2 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix}$$

Zero Determinants

Are the following determinants zero, or nonzero?

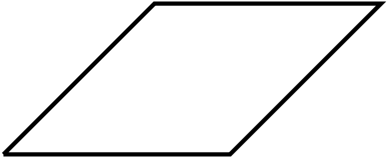
$$\det \begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \neq 0$$

$$\det \begin{bmatrix} a_1 & a_2 \\ 5a_1 & 5a_2 \end{bmatrix} = 0$$

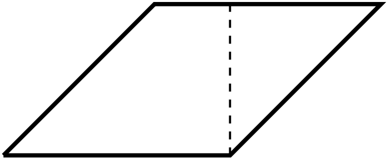
$$\det \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \end{bmatrix} = 0$$

Quick Detour: Parallelograms



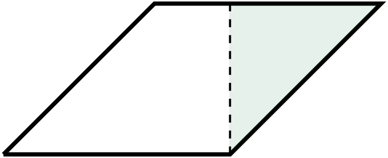
Area: (base)$\times$(height)

Quick Detour: Parallelograms



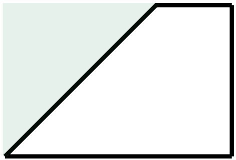
Area: (base)$\times$(height)

Quick Detour: Parallelograms



Area: (base)$\times$(height)

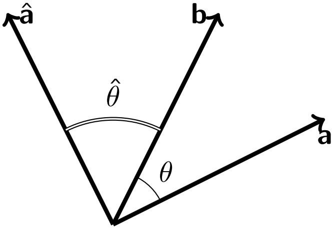
Quick Detour: Parallelograms



Area: (base)$\times$(height)

More Geometric Interpretation in Two Dimensions

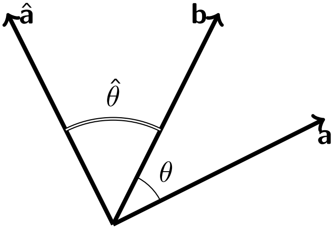
$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$



More Geometric Interpretation in Two Dimensions

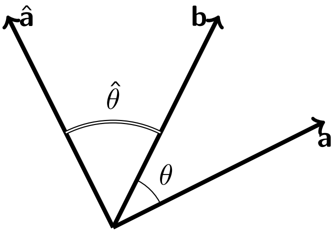
$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$

$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$



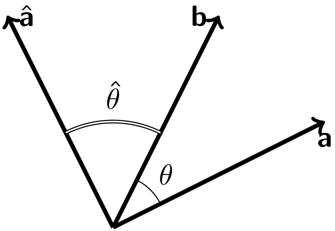
More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\pi/2 - \theta)$$



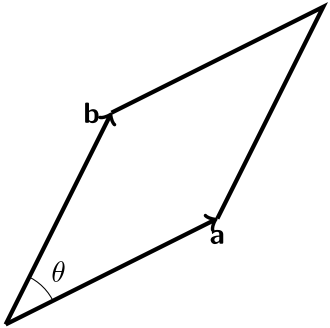
More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\pi/2 - \theta)$$
$$= \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta)$$



More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$
$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\pi/2 - \theta)$$
$$= \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta)$$



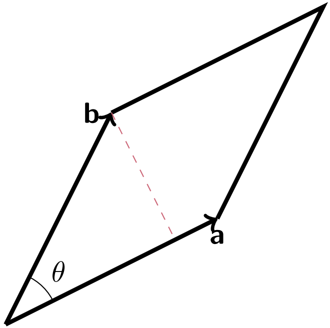
More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$

$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$

$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\pi/2 - \theta)$$

$$= \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta)$$



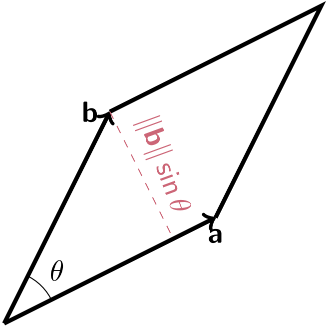
More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$

$$= \|\hat{\mathbf{a}}\|\|\mathbf{b}\| \cos(\hat{\theta})$$

$$= \|\hat{\mathbf{a}}\|\|\mathbf{b}\| \cos(\pi/2 - \theta)$$

$$= \|\mathbf{a}\|\|\mathbf{b}\| \sin(\theta)$$



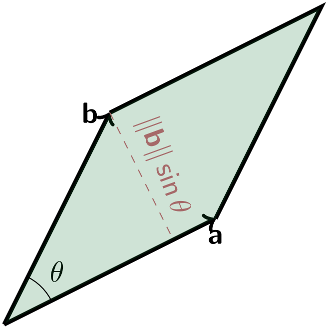
More Geometric Interpretation in Two Dimensions

$$\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = a_1 b_2 - a_2 b_1 = \begin{bmatrix} -a_2 \\ a_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \hat{\mathbf{a}} \cdot \mathbf{b}$$

$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\hat{\theta})$$

$$= \|\hat{\mathbf{a}}\| \|\mathbf{b}\| \cos(\pi/2 - \theta)$$

$$= \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta) = \text{area of parallelogram}$$



In general:

$$\left|\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\right| = \text{ area of parallelogram spanned by } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

In general:

$$\left|\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\right| = \text{ area of parallelogram spanned by } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Example: Find the area of the parallelogram with one side given by $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and the other side $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

In general:

$$\left|\det\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\right| = \text{ area of parallelogram spanned by } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Example: Find the area of the parallelogram with one side given by $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and the other side $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

$$\begin{aligned}\left|\det\begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}\right| &= |(2)(4) - (6)(-3)| \\ &= 8 + 18 = 26\end{aligned}$$

In general:

$$\left|\det\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\right| = \text{ area of parallelogram spanned by } \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Example: Find the area of the parallelogram with one side given by $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and the other side $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

$$\begin{aligned} \left|\det\begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}\right| &= |(2)(4) - (6)(-3)| \\ &= 8 + 18 = 26 \end{aligned}$$

Silly Example: Find the area of the rectangle with corners $(0, 0)$, $(x, 0)$, $(0, y)$, and (x, y).

Vector Operations

Addition: (vector)+(vector)=(vector)

Vector Operations

Addition: (vector)+(vector)=(vector)

Scalar Multiplication: (scalar)(vector)=(vector)

Vector Operations

Addition: (vector)+(vector)=(vector)

Scalar Multiplication: (scalar)(vector)=(vector)

Dot Product: (vector)$\cdot$(vector)=(scalar)

Vector Operations

Addition: (vector)$+$(vector)$=$(vector)

Scalar Multiplication: (scalar)(vector)$=$(vector)

Dot Product: (vector)$\cdot$(vector)$=$(scalar)

Cross Product: (vector)$\times$(vector)$=$(vector)

Vector Operations

Addition: (vector)+(vector)=(vector)

Scalar Multiplication: (scalar)(vector)=(vector)

Dot Product: (vector)$\cdot$(vector)=(scalar)

Cross Product: (vector)$\times$(vector)=(vector), but only in $\mathbb{R}^3$

Cross Product

ONLY defined in three dimensions.

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Cross Product

ONLY defined in three dimensions.

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Cross Product

ONLY defined in three dimensions.

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Mnemonic:

$$\det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} =$$

Cross Product

ONLY defined in three dimensions.

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Mnemonic:

$$\begin{aligned} \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} &= \mathbf{i} \det \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix} - \mathbf{j} \det \begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} + \mathbf{k} \det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \\ &= \mathbf{i}(a_2 b_3 - a_3 b_2) - \mathbf{j}(a_1 b_3 - a_3 b_1) + \mathbf{k}(a_1 b_2 - a_2 b_1) \\ &= \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \end{aligned}$$

Practice

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}0\\1\\-1\end{bmatrix} \qquad \begin{bmatrix}0\\1\\-1\end{bmatrix} \times \begin{bmatrix}1\\2\\3\end{bmatrix}$$

Practice

$$\begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}-5\\1\\1\end{bmatrix} \qquad \begin{bmatrix}0\\1\\-1\end{bmatrix} \times \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}5\\-1\\-1\end{bmatrix}$$

Practice

$$\begin{bmatrix}1\\2\\3\end{bmatrix}\times\begin{bmatrix}0\\1\\-1\end{bmatrix}=\begin{bmatrix}-5\\1\\1\end{bmatrix} \qquad \begin{bmatrix}0\\1\\-1\end{bmatrix}\times\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}5\\-1\\-1\end{bmatrix}$$

Not commutative! (But almost.)

Geometric Interpretation

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and to $\mathbf{b}$.

Geometric Interpretation

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and to $\mathbf{b}$.

   Verify:

$$\begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0$$

Geometric Interpretation

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and to $\mathbf{b}$.

   Verify:

$$\begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0$$

2. $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$,
   where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, $0 \leq \theta \leq \pi$.

Geometric Interpretation

1. $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and to $\mathbf{b}$.

   Verify:

$$\begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0$$

2. $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin\theta$,
   where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, $0 \leq \theta \leq \pi$.
   Thus, $\sin\theta$ is positive, and $\|\mathbf{a} \times \mathbf{b}\|$ is the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.

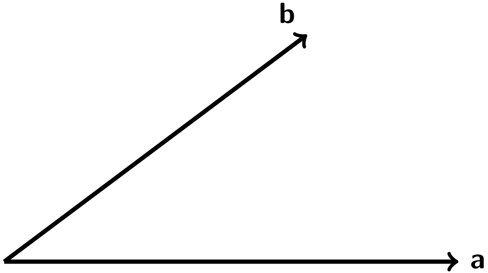
Geometric Interpretation

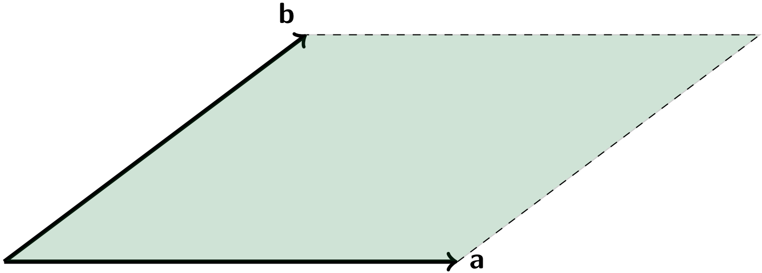
1. **a** $\times$ **b** is orthogonal to **a** and to **b** .

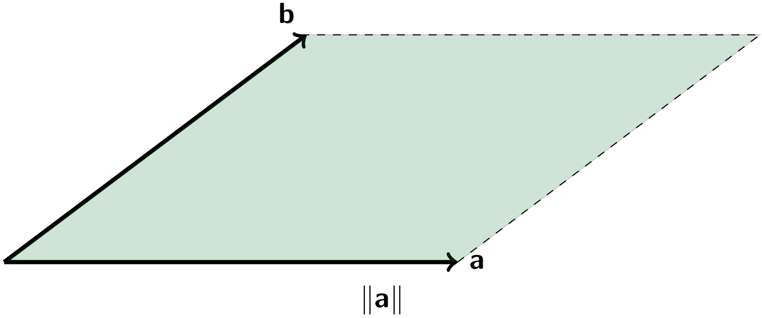
   Verify:

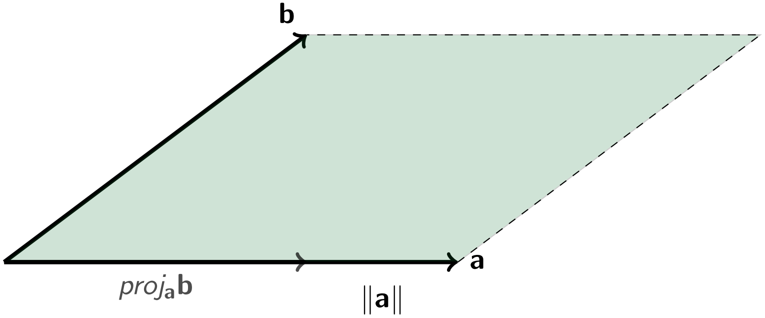
$$\begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0$$

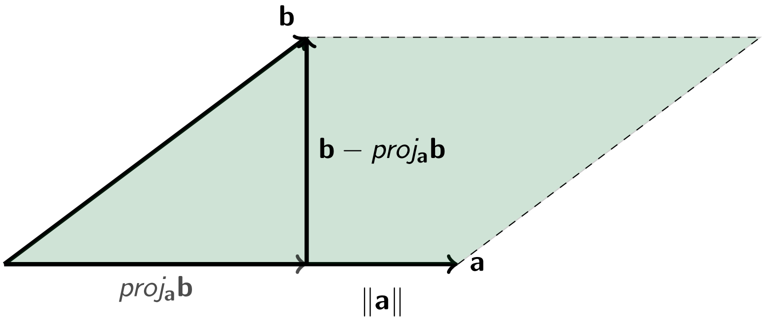
2. $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin\theta$,
   where θ is the angle between **a** and **b** , $0 \leq \theta \leq \pi$.
   Thus, $\sin\theta$ is positive, and $\|\mathbf{a} \times \mathbf{b}\|$ is the area of the parallelogram spanned by **a** and **b** .
3. The vectors **a** , **b** , and **a** $\times$ **b** obey the *right hand rule*. That is, if you curl your fingers towards your palm from **a** to **b** , your thumb points in the direction of **a** $\times$ **b**.

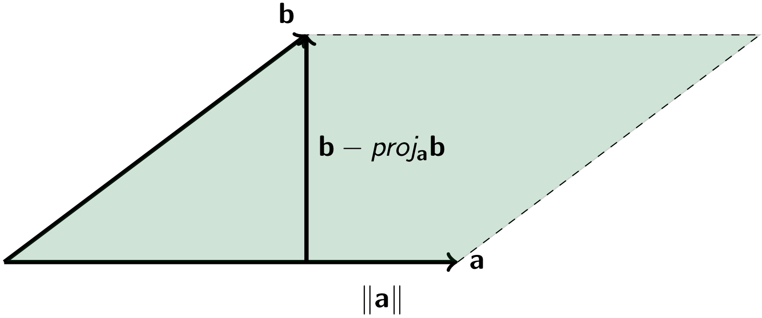

b
a


b
a


b
a
$\|\mathbf{a}\|$


b
a
$proj_{\mathbf{a}}\mathbf{b}$
$\|\mathbf{a}\|$


$\mathbf{b}$
$\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}$
$\mathbf{a}$
$proj_{\mathbf{a}}\mathbf{b}$
$\|\mathbf{a}\|$



$$A = (\text{base})(\text{height}) = \|\mathbf{a}\|\|\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}\|$$

$\|\mathbf{a} \times \mathbf{b}\|$ = area of parallelogram

$$A^2 = \|\mathbf{a}\|^2 \|\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}\|^2$$

$\|\mathbf{a} \times \mathbf{b}\|$ = area of parallelogram

$$\begin{aligned} A^2 &= \|\mathbf{a}\|^2 \|\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}\|^2 \\ &= \|\mathbf{a}\|^2 \left\| \mathbf{b} - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) \mathbf{a} \right\|^2 \end{aligned}$$

$\|\mathbf{a} \times \mathbf{b}\|$ = area of parallelogram

$$
\begin{aligned}
A^2 &= \|\mathbf{a}\|^2 \|\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}\|^2 \\
&= \|\mathbf{a}\|^2 \left\| \mathbf{b} - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) \mathbf{a} \right\|^2 \\
&= \|\mathbf{a}\|^2 \left(\mathbf{b} \cdot \mathbf{b} - 2(\mathbf{b} \cdot \mathbf{a}) \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) + \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right)^2 \|\mathbf{a}\|^2 \right) \\
&= \|\mathbf{a}\|^2 \left(\|\mathbf{b}\|^2 - \frac{(\mathbf{a} \cdot \mathbf{b})^2}{\|\mathbf{a}\|^2} \right)
\end{aligned}
$$

$\|\mathbf{a} \times \mathbf{b}\|$ = area of parallelogram

$$
\begin{aligned}
A^2 &= \|\mathbf{a}\|^2 \|\mathbf{b} - proj_{\mathbf{a}}\mathbf{b}\|^2 \\
&= \|\mathbf{a}\|^2 \left\| \mathbf{b} - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) \mathbf{a} \right\|^2 \\
&= \|\mathbf{a}\|^2 \left(\mathbf{b} \cdot \mathbf{b} - 2(\mathbf{b} \cdot \mathbf{a}) \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) + \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right)^2 \|\mathbf{a}\|^2 \right) \\
&= \|\mathbf{a}\|^2 \left(\|\mathbf{b}\|^2 - \frac{(\mathbf{a} \cdot \mathbf{b})^2}{\|\mathbf{a}\|^2} \right) \\
&= \|\mathbf{a}\|^2 \|\mathbf{b}\|^2 - (\mathbf{a} \cdot \mathbf{b})^2 \\
&= (a_1^2 + a_2^2 + a_3^2)^2 (b_1^2 + b_2^2 + b_3^2)^2 - (a_1 b_1 + a_2 b_2 + a_3 b_3)^2 \\
&= \cdots = (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2 + (a_1 b_2 - a_2 b_1)^2 \\
&= \|\mathbf{a} \times \mathbf{b}\|^2
\end{aligned}
$$

Find the Area of the Parallelograms

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\1\\-2\end{bmatrix}$.

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}4\\3\end{bmatrix}$.

Find the Area of the Parallelograms

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\1\\-2\end{bmatrix}$.

$$\left\| \begin{bmatrix}1\\1\\1\end{bmatrix} \times \begin{bmatrix}3\\1\\-2\end{bmatrix} \right\| = \sqrt{38}$$

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}4\\3\end{bmatrix}$.

Find the Area of the Parallelograms

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\1\\-2\end{bmatrix}$.

$$\left\| \begin{bmatrix}1\\1\\1\end{bmatrix} \times \begin{bmatrix}3\\1\\-2\end{bmatrix} \right\| = \sqrt{38}$$

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}4\\3\end{bmatrix}$.

$$\left| \det \begin{bmatrix}1 & 2\\4 & 3\end{bmatrix} \right| = 5$$

Find the Area of the Parallelograms

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}3\\1\\-2\end{bmatrix}$.

$$\left\| \begin{bmatrix}1\\1\\1\end{bmatrix} \times \begin{bmatrix}3\\1\\-2\end{bmatrix} \right\| = \sqrt{38}$$

Find the area of the parallelogram spanned by $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}4\\3\end{bmatrix}$.

$$\left| \det \begin{bmatrix}1 & 2\\4 & 3\end{bmatrix} \right| = 5$$

Can you do that with a cross product, by imagining these vectors in $\mathbb{R}^3$?

Suppose a plane contains the points $P_1(3,2,2)$, $P_2(2,2,1)$, and $P_3(1,1,1)$. Find a normal vector to the plane. That is, find a vector that is perpendicular to every line on the plane.

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = \quad \mathbf{b} \times \mathbf{a}$

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$

Properties of Cross Product (p. 31)

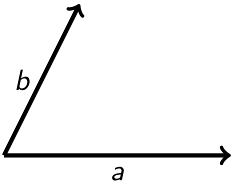
1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
   https://proofwiki.org/wiki/Lagrange's_Formula

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

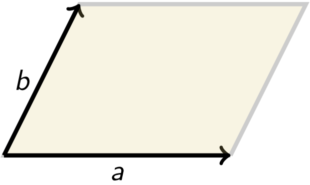
Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$



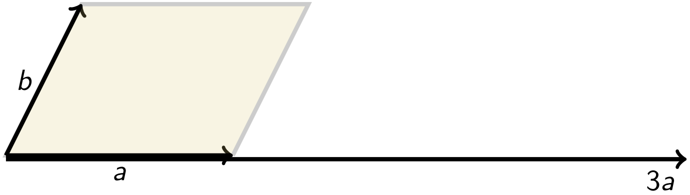
Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$



Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$



Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

4. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

4. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

Properties of Cross Product (p. 31)

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

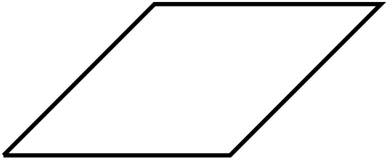
4. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
   Is it also true that $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$?

Properties of Cross Product (p. 31)

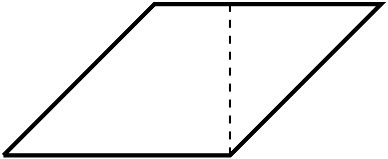
1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$
3. $s(\mathbf{a} \times \mathbf{b}) = (s\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (s\mathbf{b})$

4. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ "triple product"

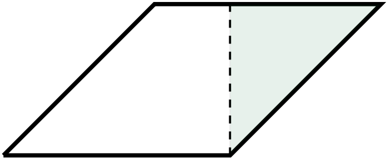
Parallelograms



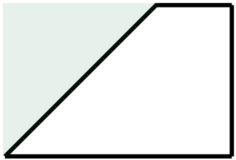
Parallelograms



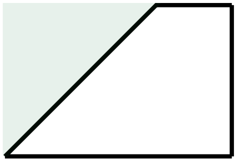
Parallelograms



Parallelograms

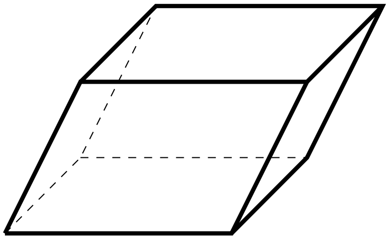


Parallelograms

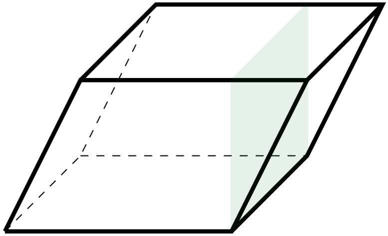


Area: (base)$\times$(height)

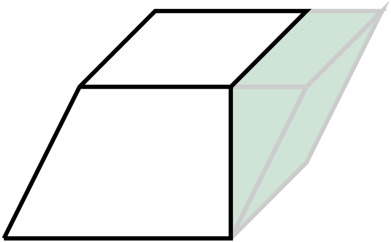
Parallelepipeds



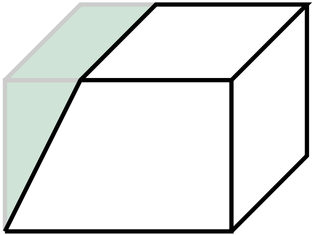
Parallelepipeds



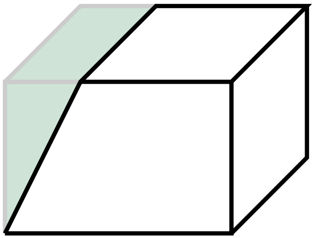
Parallelepipeds



Parallelepipeds

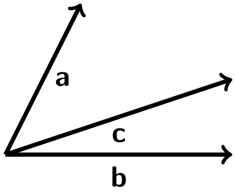


Parallelepipeds

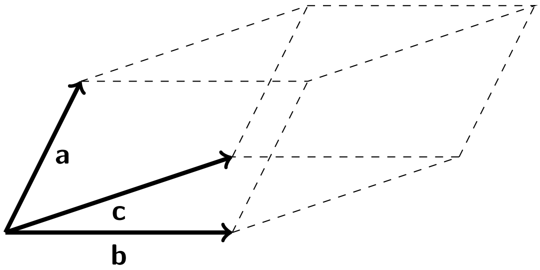


Volume: (area of base)$\times$(height)

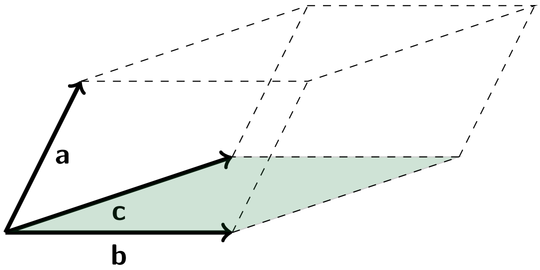
Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



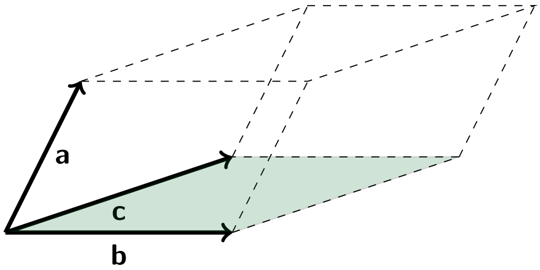
Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

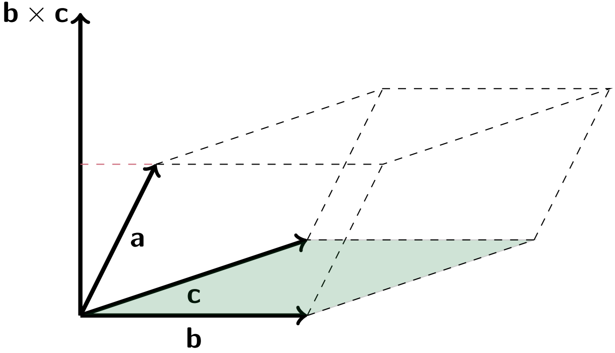


Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



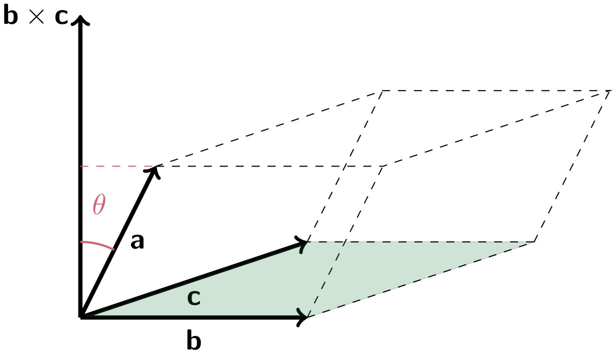
Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



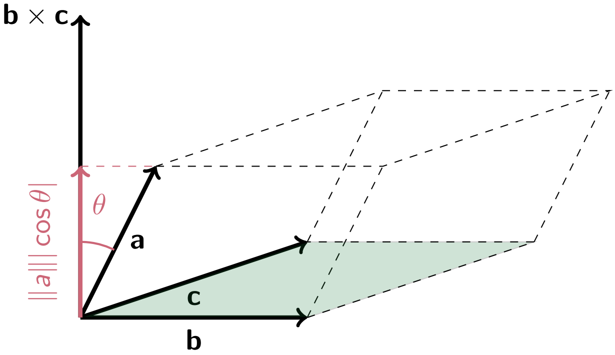
Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



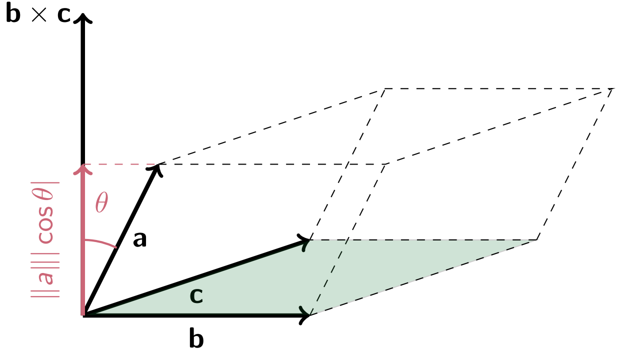
Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

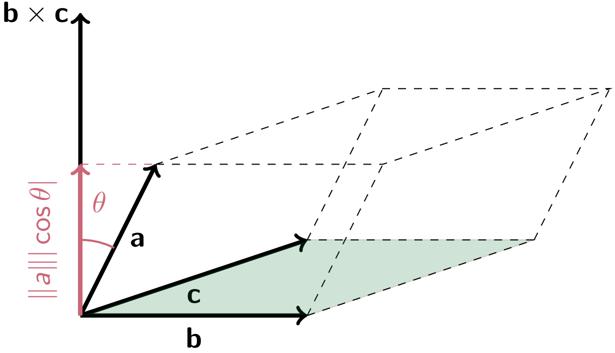
Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

Height of parallelepiped: $\|\mathbf{a}\|| \cos\theta|$

Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



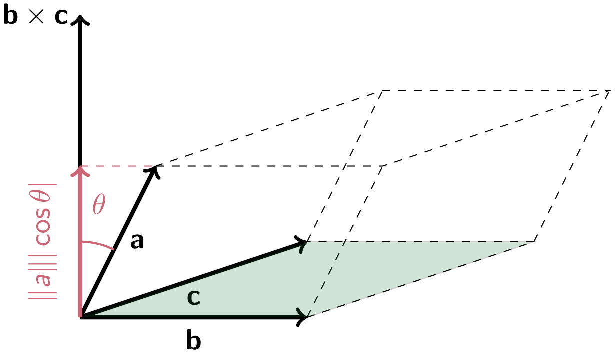
Area of base: $\|\mathbf{b} \times \mathbf{c}\|$

Height of parallelepiped: $\|\mathbf{a}\| |\cos\theta|$

Volume of parallelepiped:

(area of base)(height)$= \|\mathbf{a}\| \|\mathbf{b} \times \mathbf{c}\| |\cos\theta| =$

Triple Product: $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$



Area of base: $\|\mathbf{b} \times \mathbf{c}\|$
Height of parallelepiped: $\|\mathbf{a}\|| \cos\theta|$

Volume of parallelepiped:
(area of base)(height)$= \|\mathbf{a}\|\|\mathbf{b} \times \mathbf{c}\|| \cos\theta| = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$

Calculating the Triple Product

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) =$$

Calculating the Triple Product

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

Calculating the Triple Product

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \mathbf{a} \cdot \begin{bmatrix} \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} \\ -\det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} \\ \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \end{bmatrix}$$

Calculating the Triple Product

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot \det \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \mathbf{a} \cdot \begin{bmatrix} \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} \\ -\det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} \\ \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \end{bmatrix}$$

$$= a_1 \det \begin{bmatrix} b_2 & b_3 \\ c_2 & c_3 \end{bmatrix} - a_2 \det \begin{bmatrix} b_1 & b_3 \\ c_1 & c_3 \end{bmatrix} + a_3 \det \begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$$

Calculating the Triple Product

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{a}\cdot\det\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix} = \mathbf{a}\cdot\begin{bmatrix}\det\begin{bmatrix}b_2 & b_3\\ c_2 & c_3\end{bmatrix}\\ -\det\begin{bmatrix}b_1 & b_3\\ c_1 & c_3\end{bmatrix}\\ \det\begin{bmatrix}b_1 & b_2\\ c_1 & c_2\end{bmatrix}\end{bmatrix}$$

$$= a_1\det\begin{bmatrix}b_2 & b_3\\ c_2 & c_3\end{bmatrix} - a_2\det\begin{bmatrix}b_1 & b_3\\ c_1 & c_3\end{bmatrix} + a_3\det\begin{bmatrix}b_1 & b_2\\ c_1 & c_2\end{bmatrix}$$

$$= \det\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$$

Find the volume of the parallelepiped spanned by:
$\begin{bmatrix}0\\0\\1\end{bmatrix}$, $\begin{bmatrix}1\\-1\\1\end{bmatrix}$, and $\begin{bmatrix}-1\\-1\\1\end{bmatrix}$.

Find the volume of the parallelepiped spanned by:
$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.

For positive a, b, and c, find the determinant and interpret it as a volume:

$$\det \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Find the volume of the parallelepiped spanned by:
$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.

For positive a, b, and c, find the determinant and interpret it as a volume:

$$\det \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Calculate and explain geometrically:

$$\det \begin{bmatrix} 2 & 0 & 3 \\ 8 & 1 & 7 \\ 20 & 3 & 15 \end{bmatrix}$$

Right-hand rule!
Curl your fingers FROM $\mathbf{a}$ TO $\mathbf{b}$. Your thumb points in the direction of the vector $\mathbf{a} \times \mathbf{b}$.

Right-Hand Rule

Predict the following cross products without using the cross-product calculation. Draw your results. Check using the cross-product calculation.

$$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix}$$

Given any 3-dimensional vector $\mathbf{a}$, is there a simple expression for $\mathbf{a} \times \mathbf{a}$?

Given any 3-dimensional vector $\mathbf{a}$, is there a simple expression for $\mathbf{a} \times \mathbf{a}$?

What about $(s\mathbf{a}) \times \mathbf{a}$ for a scalar s?

Given any 3-dimensional vector $\mathbf{a}$, is there a simple expression for $\mathbf{a} \times \mathbf{a}$?

What about $(s\mathbf{a}) \times \mathbf{a}$ for a scalar s?

What about $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$?

Given any 3-dimensional vector $\mathbf{a}$, is there a simple expression for $\mathbf{a} \times \mathbf{a}$?

What about $(s\mathbf{a}) \times \mathbf{a}$ for a scalar s?

What about $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$?

Consider $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$. Will this vector be in the same plane as $\mathbf{b}$ and $\mathbf{c}$, or in an orthogonal plane?

Given any 3-dimensional vector $\mathbf{a}$, is there a simple expression for $\mathbf{a} \times \mathbf{a}$?

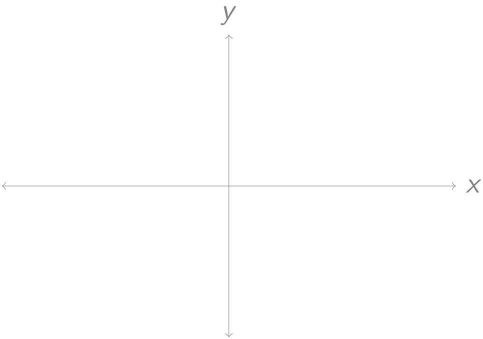
What about $(s\mathbf{a}) \times \mathbf{a}$ for a scalar s?

What about $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$?

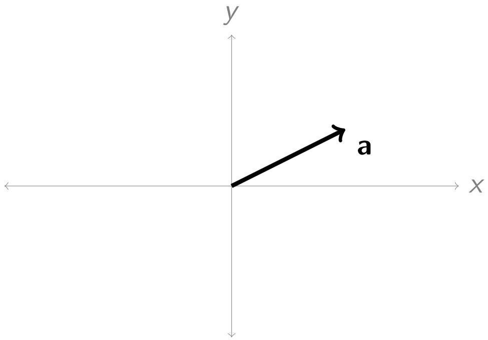
Consider $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$. Will this vector be in the same plane as $\mathbf{b}$ and $\mathbf{c}$, or in an orthogonal plane?

Notice $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c}$:
a linear combination of $\mathbf{b}$ and $\mathbf{c}$.

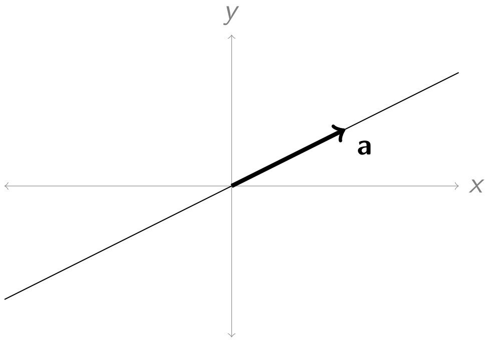
Parametric Equations of Lines



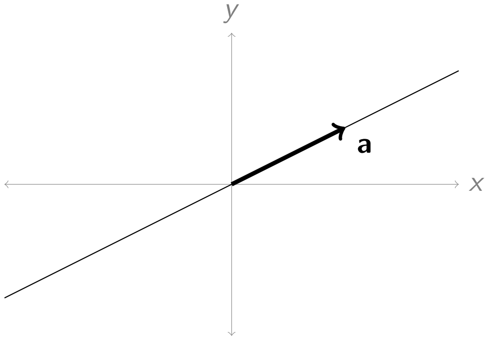
Parametric Equations of Lines



Parametric Equations of Lines



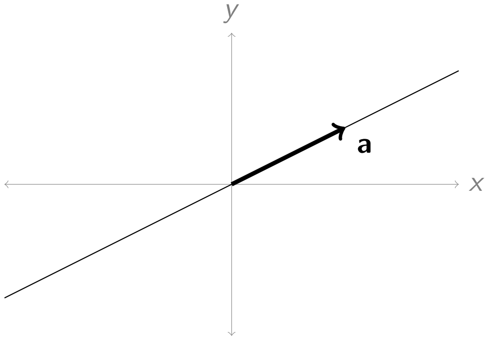
Parametric Equations of Lines



$$\mathbf{x} = s\mathbf{a}$$

Meaning: the collection of all vectors $\mathbf{x}$ that can be generated by multiplying $\mathbf{a}$ with a scalar, interpreted as points.

Parametric Equations of Lines

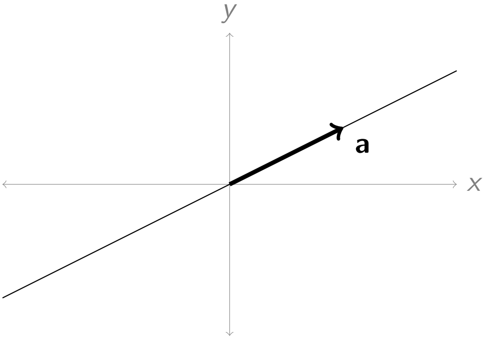


Line passing through the origin:

$$\mathbf{x} = s\mathbf{a}$$

Meaning: the collection of all vectors **x** that can be generated by multiplying **a** with a scalar, interpreted as points.

Parametric Equations of Lines

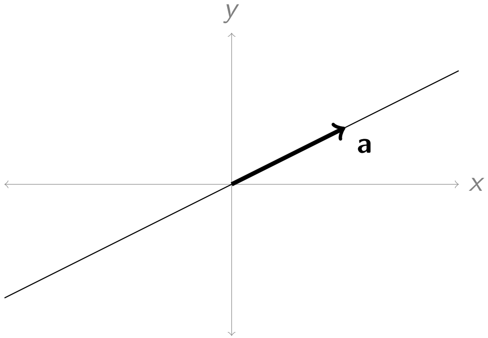


Line passing through the origin:

$$\mathbf{x} = s\mathbf{a}$$

Question: is this the only such equation for the line?

Parametric Equations of Lines

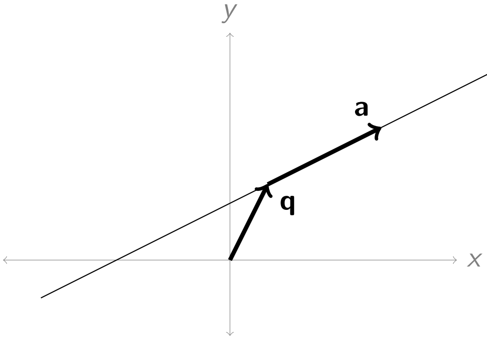


Line passing through the origin:

$$\mathbf{x} = s\mathbf{a}$$

Can we use this equation with a line *not* through the origin?

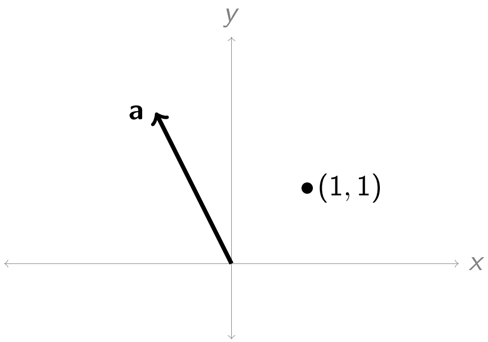
Parametric Equations of Lines



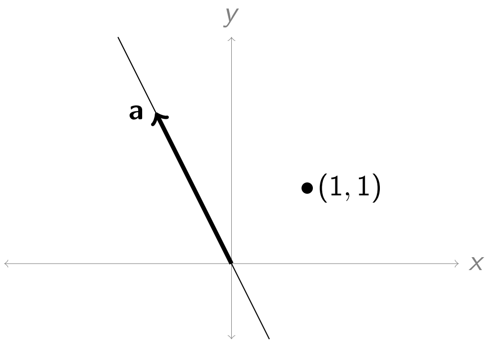
General equation of a line:

$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$

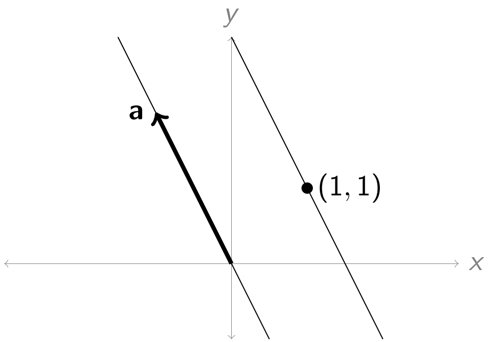
Where we mean: the line consists of all points $\mathbf{x}$ that can be written this way for some scalar s.



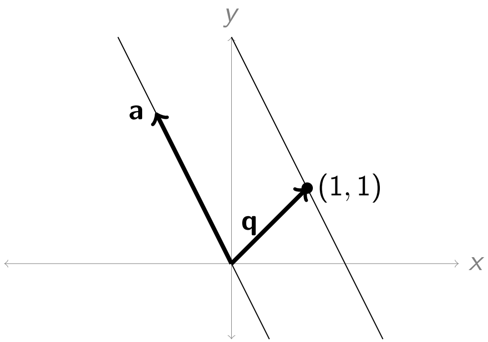
Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$.



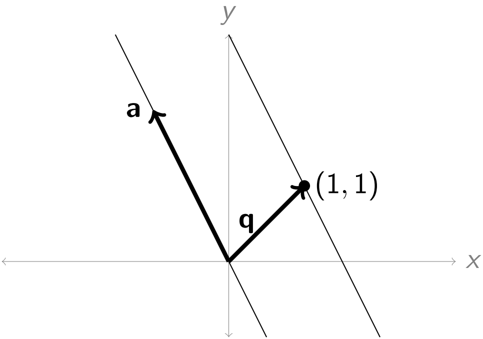
Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$.



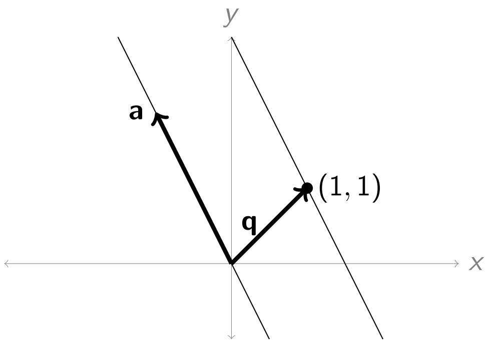
Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$.



Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$.

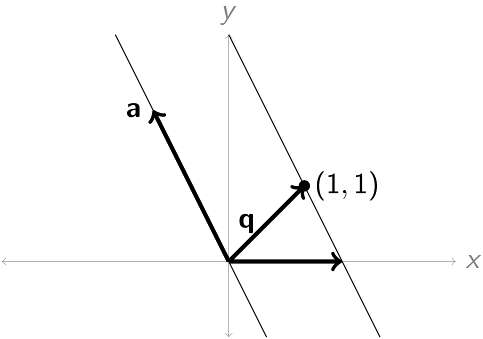


Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$



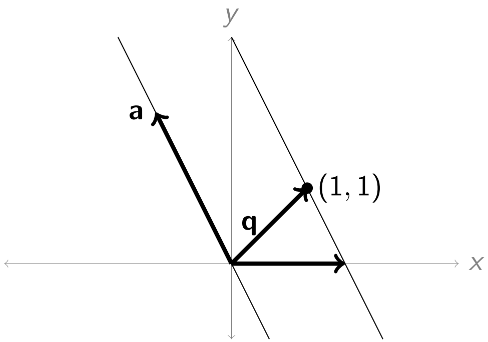
Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Can you find another?



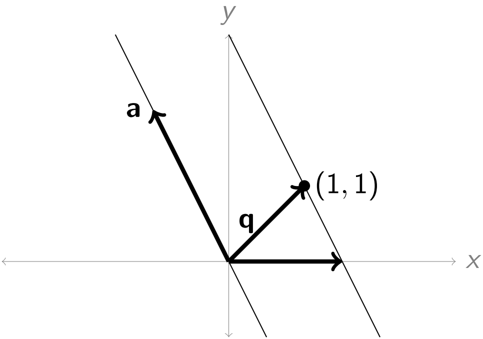
Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Can you find another?



Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

Can you find another? $\mathbf{x} = \begin{bmatrix} 3/2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

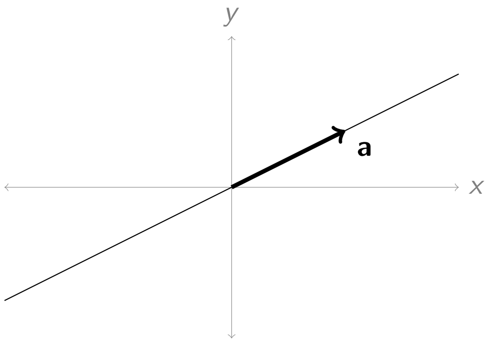


Find a parametric equation describing the line in the direction of $\mathbf{a} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, passing through the point $(1, 1)$. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

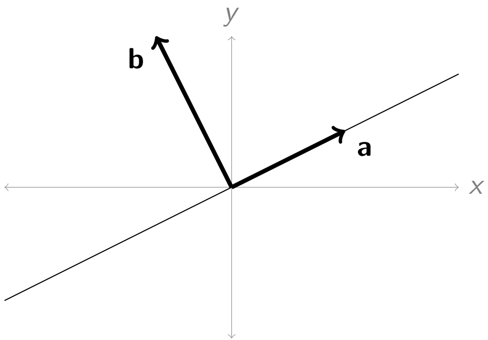
Can you find another? $\mathbf{x} = \begin{bmatrix} 3/2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\mathbf{x} = \begin{bmatrix} 3/2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

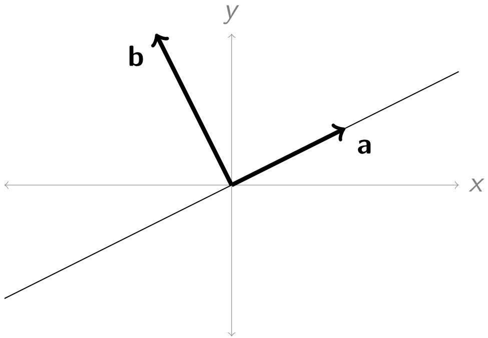
Component Equations of Lines



Component Equations of Lines

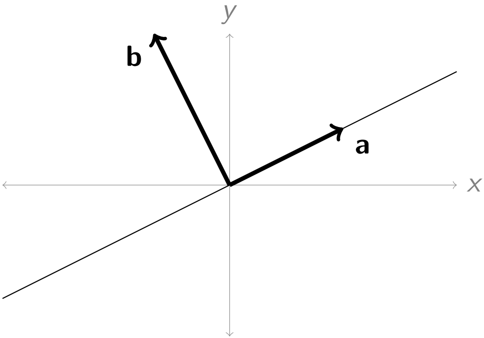


Component Equations of Lines



Line passing through the origin:

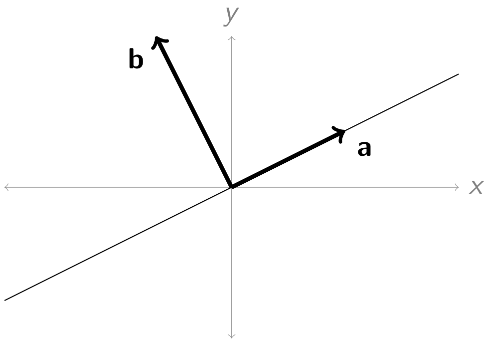
Component Equations of Lines



Line passing through the origin:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Component Equations of Lines

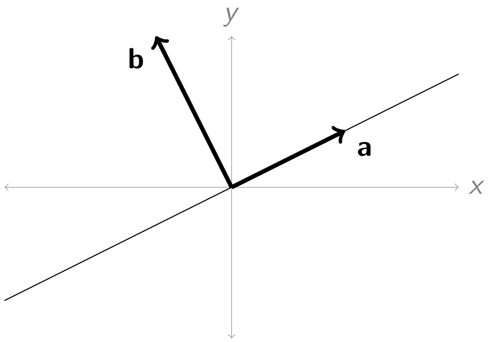


Line passing through the origin:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\Rightarrow \boxed{b_1 x + b_2 y = 0}$$

Component Equations of Lines

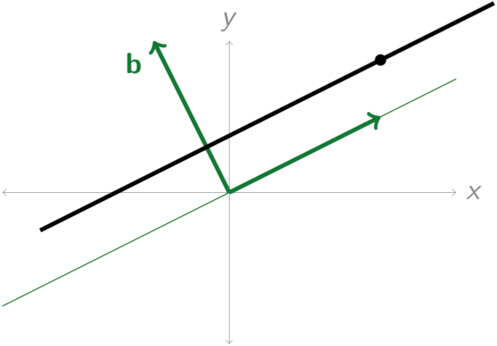


Line passing through the origin:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

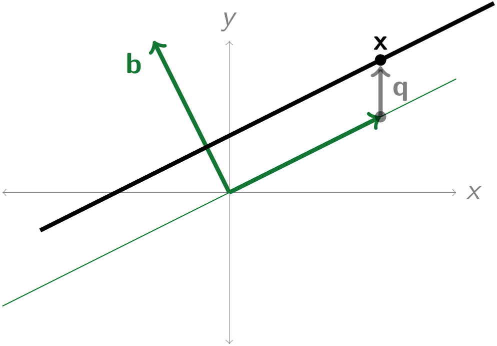
$$\Rightarrow \boxed{b_1 x + b_2 y = 0} \qquad \Rightarrow y = (-b_2/b_1)x$$

Components Equations of Lines



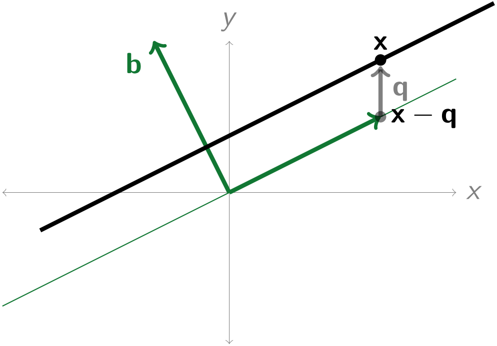
General Line in $\mathbb{R}^2$

Components Equations of Lines



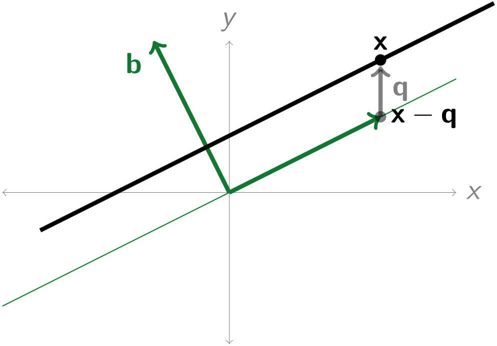
General Line in $\mathbb{R}^2$

Components Equations of Lines



General Line in $\mathbb{R}^2$

Components Equations of Lines



General Line in $\mathbb{R}^2$

$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0$$

$$\mathbf{x} \cdot \mathbf{b} = \mathbf{q} \cdot \mathbf{b}$$

$$\boxed{b_1 x + b_2 y = c}$$

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} x = 3 + s \\ y = -1 + s \end{cases}$$

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} x = 3 + s \\ y = -1 + s \end{cases}$$

$$\Leftrightarrow \begin{cases} s = x - 3 \\ s = y + 1 \end{cases}$$

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

$$\begin{aligned}
&\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
\Leftrightarrow &\begin{cases} x = 3 + s \\ y = -1 + s \end{cases} \\
\Leftrightarrow &\begin{cases} s = x - 3 \\ s = y + 1 \end{cases} \\
\Rightarrow & x - 3 = y + 1 \\
\Leftrightarrow & x - y = 4
\end{aligned}$$

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

$$\begin{aligned}
&\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
\Leftrightarrow &\begin{cases} x = 3 + s \\ y = -1 + s \end{cases} \\
\Leftrightarrow &\begin{cases} s = x - 3 \\ s = y + 1 \end{cases} \\
\Rightarrow &x - 3 = y + 1 \\
\Leftrightarrow &x - y = 4
\end{aligned}$$

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

Suppose the parametric equation of a line is given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} + s \begin{bmatrix} 2 \\ 7 \end{bmatrix}$. Convert this to an equation of the form $ax + by = c$.

$$\begin{cases} x = 3 + 2s \\ y = -1 + 7s \end{cases}$$

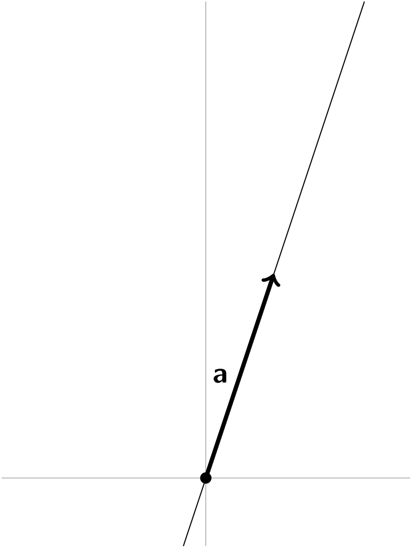
$$\Leftrightarrow \begin{cases} x - 3 = 2s \\ y + 1 = 7s \end{cases}$$

$$\Leftrightarrow \begin{cases} 7x - 21 = 14s \\ 2y + 2 = 14s \end{cases}$$

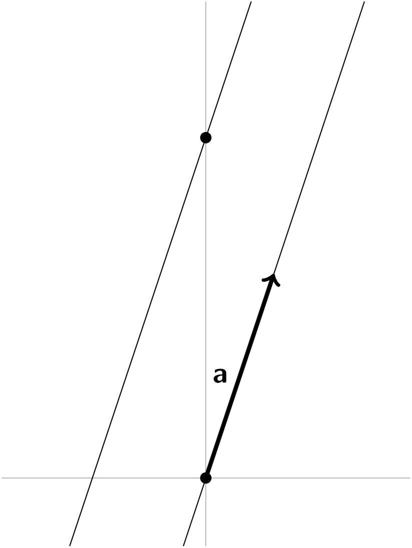
$$\Rightarrow 7x - 12 = 2y + 2 \qquad \Leftrightarrow 7x - 2y = 23$$

Give a parametric equation for the line $y = 3x + 5$.

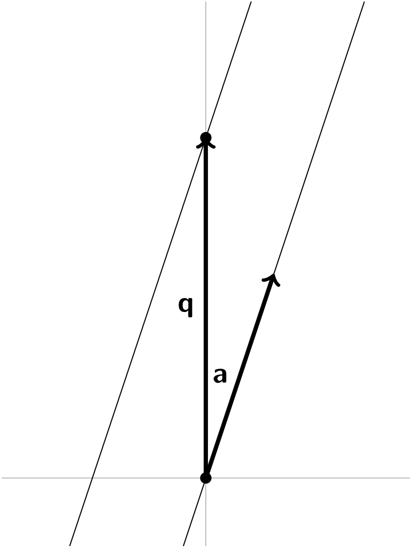
Give a parametric equation for the line $y = 3x + 5$.



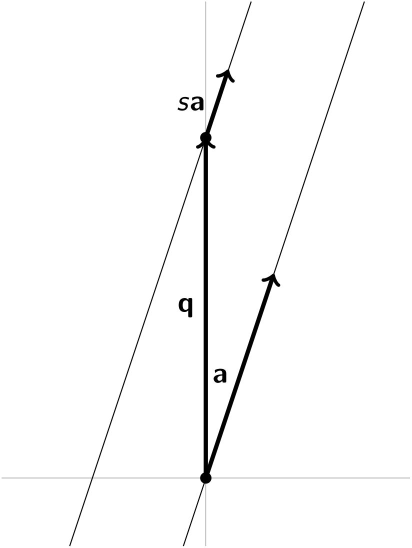
Give a parametric equation for the line $y = 3x + 5$.



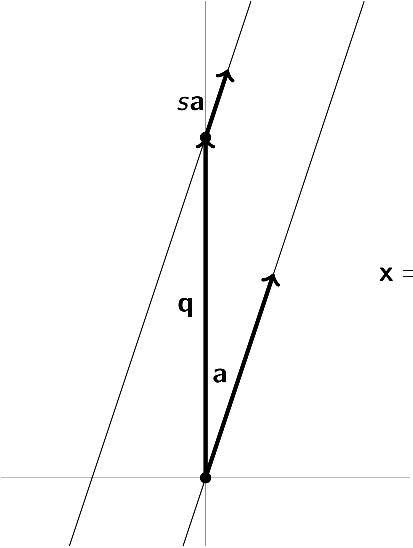
Give a parametric equation for the line $y = 3x + 5$.



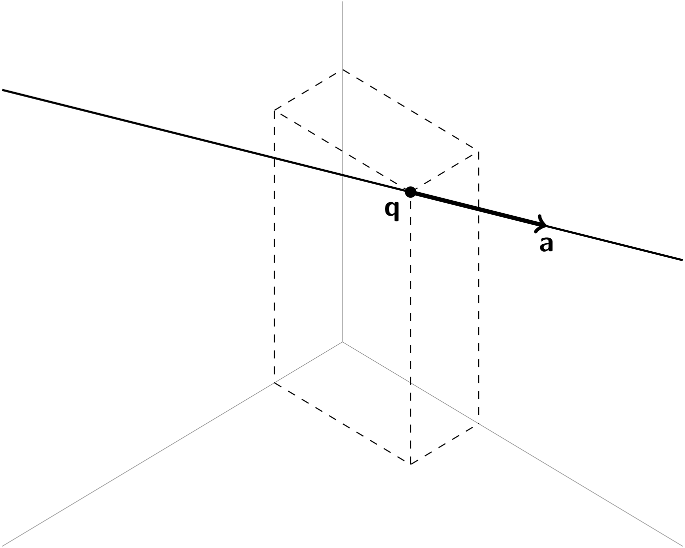
Give a parametric equation for the line $y = 3x + 5$.

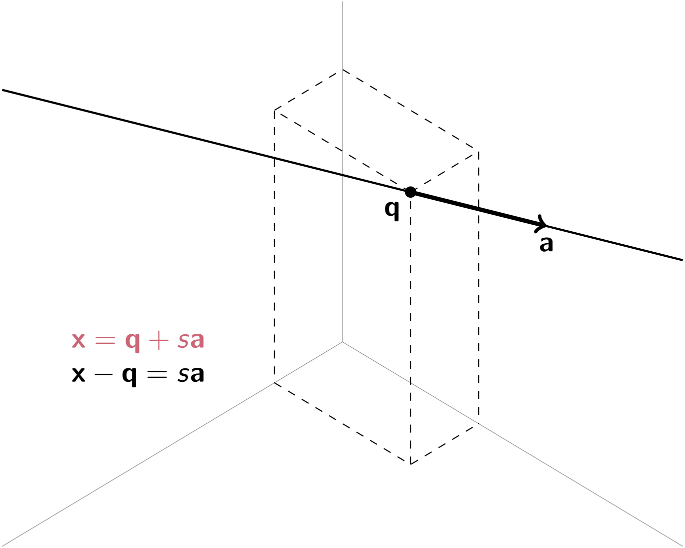


Give a parametric equation for the line $y = 3x + 5$.



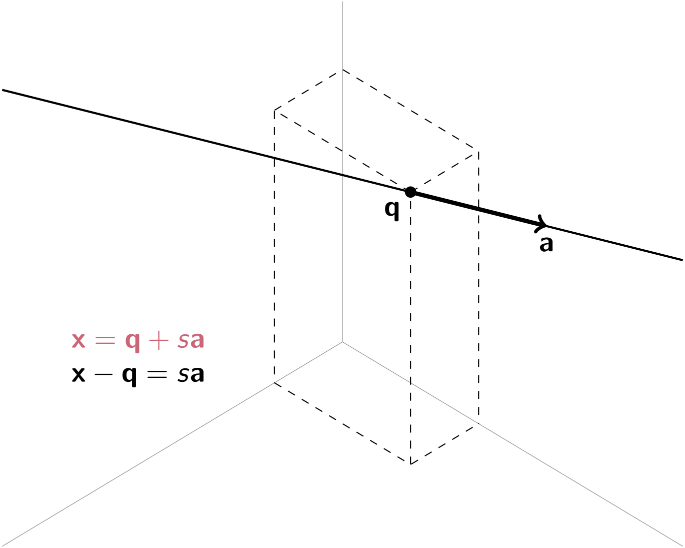
$$\mathbf{x} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} + s \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$


q
a



$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$

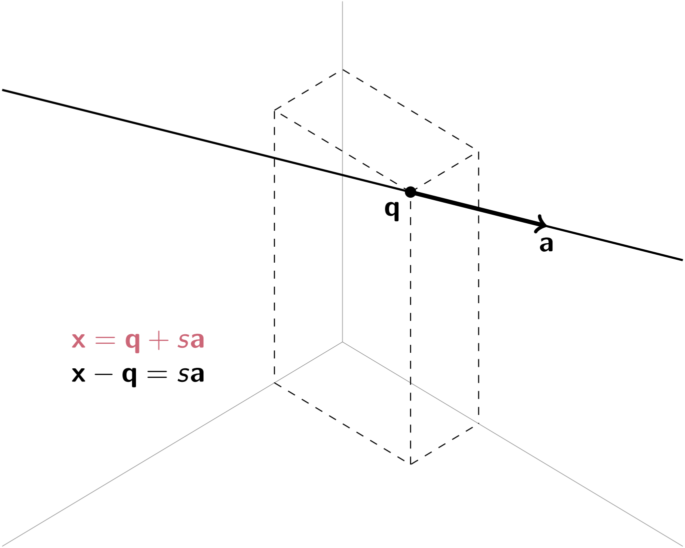
$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$



$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$

$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$

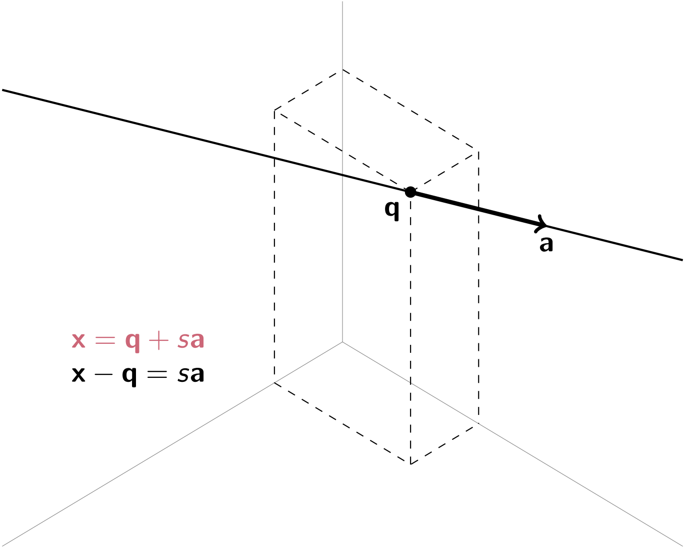
How many components do the vectors have?



$\mathbf{x} = \mathbf{q} + s\mathbf{a}$

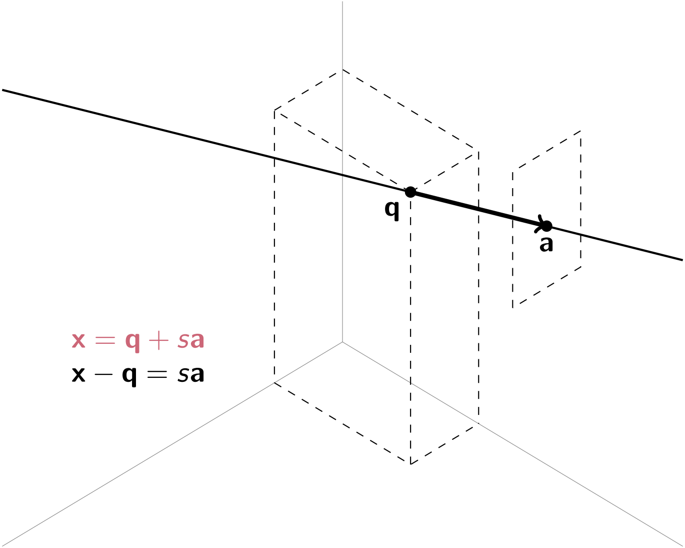
$\mathbf{x} - \mathbf{q} = s\mathbf{a}$

How many dimensions does a vector have?

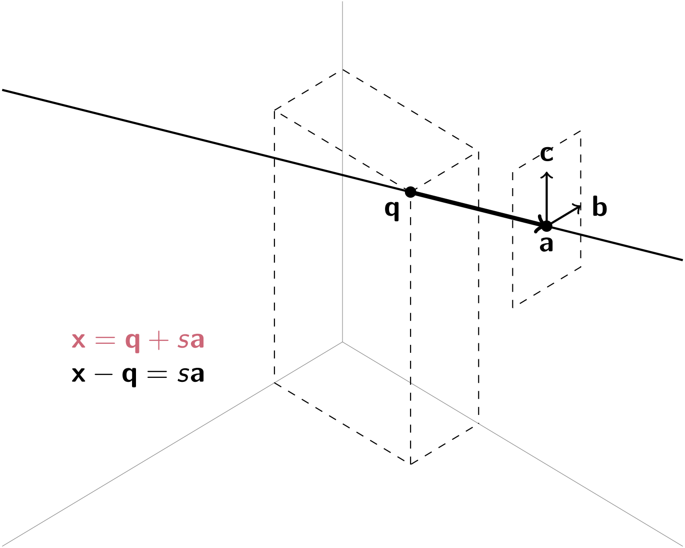


$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$

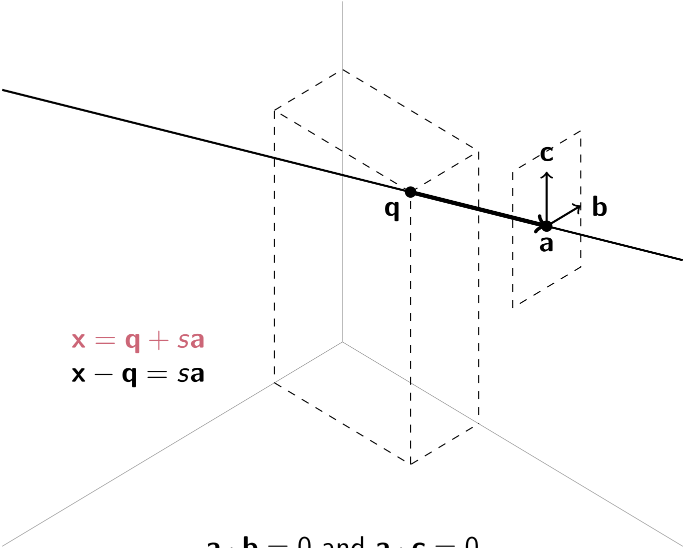
$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$



$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$
$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$



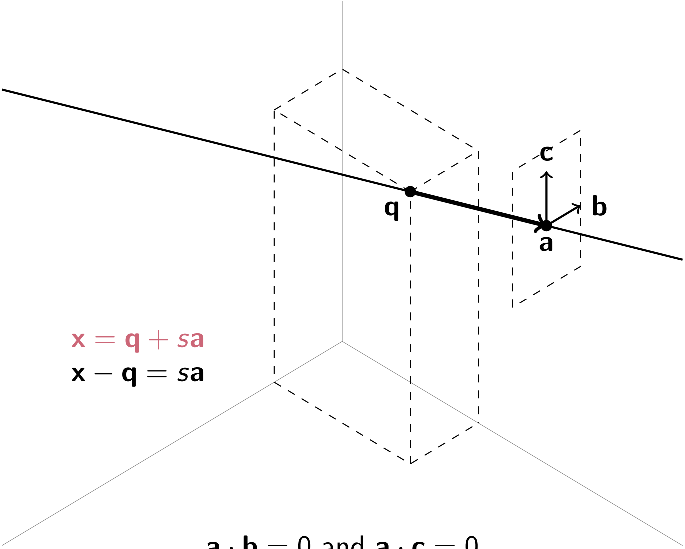
$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$
$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$



$\mathbf{x} = \mathbf{q} + s\mathbf{a}$

$\mathbf{x} - \mathbf{q} = s\mathbf{a}$

$\mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a} \cdot \mathbf{c} = 0$



$$\mathbf{x} = \mathbf{q} + s\mathbf{a}$$
$$\mathbf{x} - \mathbf{q} = s\mathbf{a}$$

$$\mathbf{a} \cdot \mathbf{b} = 0 \text{ and } \mathbf{a} \cdot \mathbf{c} = 0$$
$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0 \text{ and } (\mathbf{x} - \mathbf{q}) \cdot \mathbf{c} = 0$$

Equation of a Line in $\mathbb{R}^3$

$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0 \text{ and } (\mathbf{x} - \mathbf{q}) \cdot \mathbf{c} = 0$$

Equation of a Line in $\mathbb{R}^3$

$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0 \text{ and } (\mathbf{x} - \mathbf{q}) \cdot \mathbf{c} = 0$$

$$\mathbf{x} \cdot \mathbf{b} = \mathbf{q} \cdot \mathbf{b} \text{ and } \mathbf{x} \cdot \mathbf{c} = \mathbf{q} \cdot \mathbf{c}$$

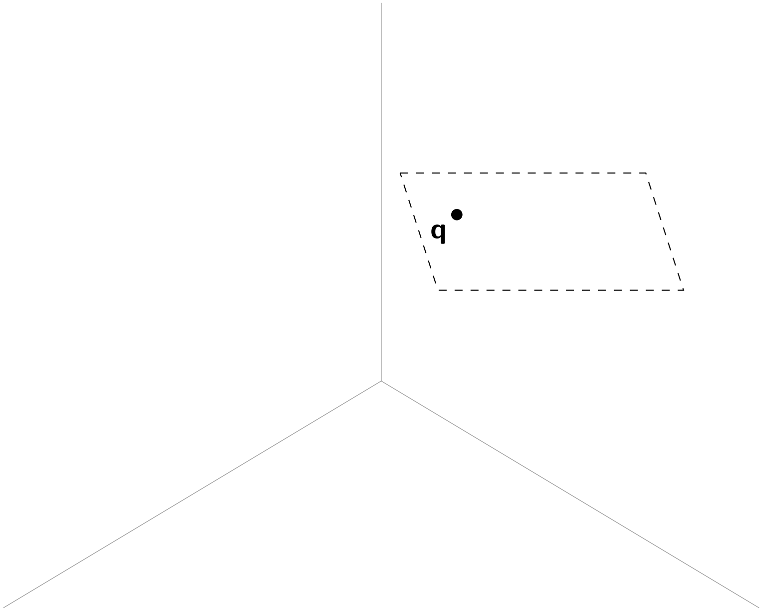
Equation of a Line in $\mathbb{R}^3$

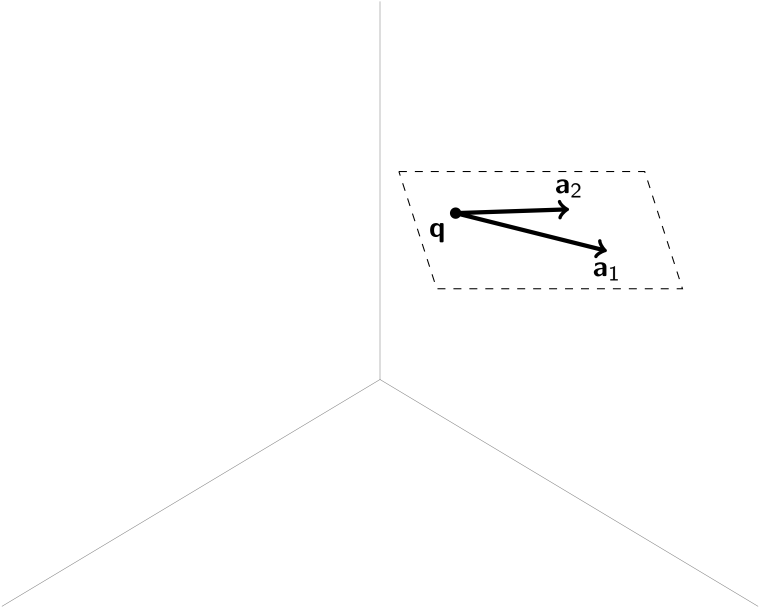
$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0 \text{ and } (\mathbf{x} - \mathbf{q}) \cdot \mathbf{c} = 0$$

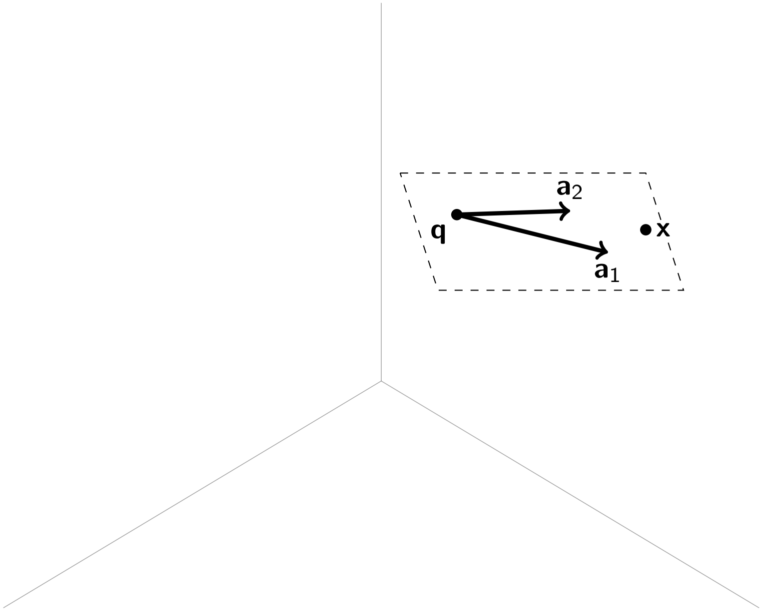
$$\mathbf{x} \cdot \mathbf{b} = \mathbf{q} \cdot \mathbf{b} \text{ and } \mathbf{x} \cdot \mathbf{c} = \mathbf{q} \cdot \mathbf{c}$$

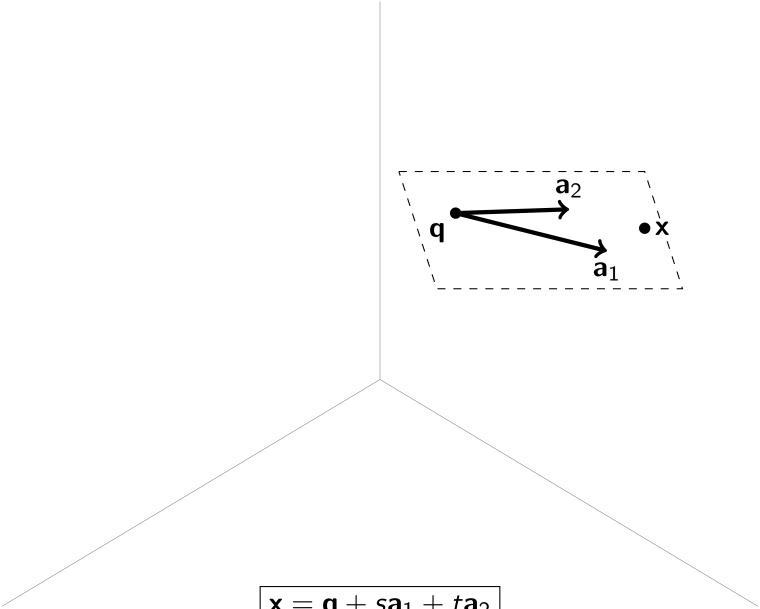
To define a line in $\mathbb{R}^3$, we need a *system* of equations:

$$\begin{cases} xb_1 + yb_2 + zb_3 & = & s_1 \\ xc_1 + yc_2 + zc_3 & = & s_2 \end{cases}$$

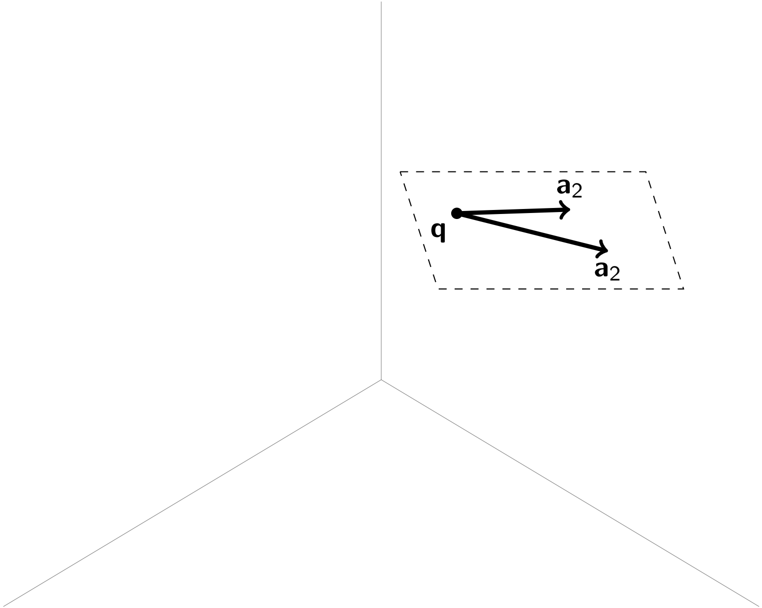

q

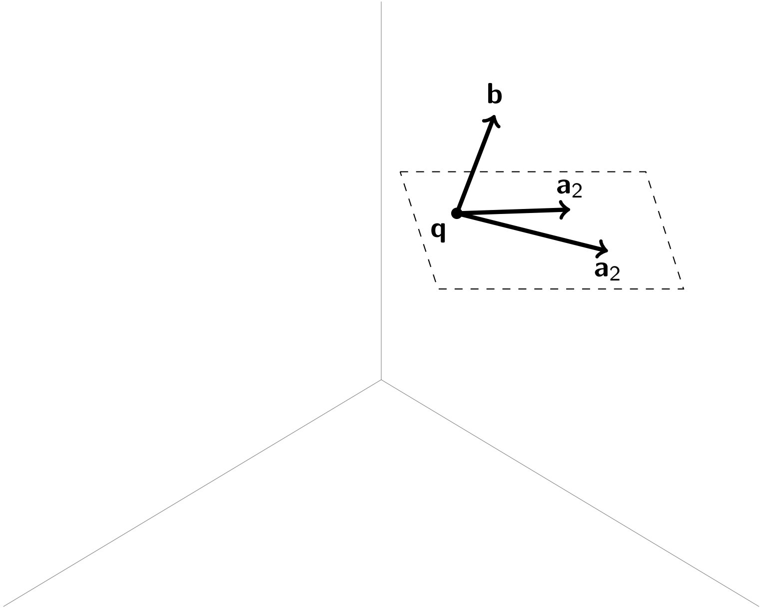

$\mathbf{a}_2$
$\mathbf{q}$
$\mathbf{a}_1$

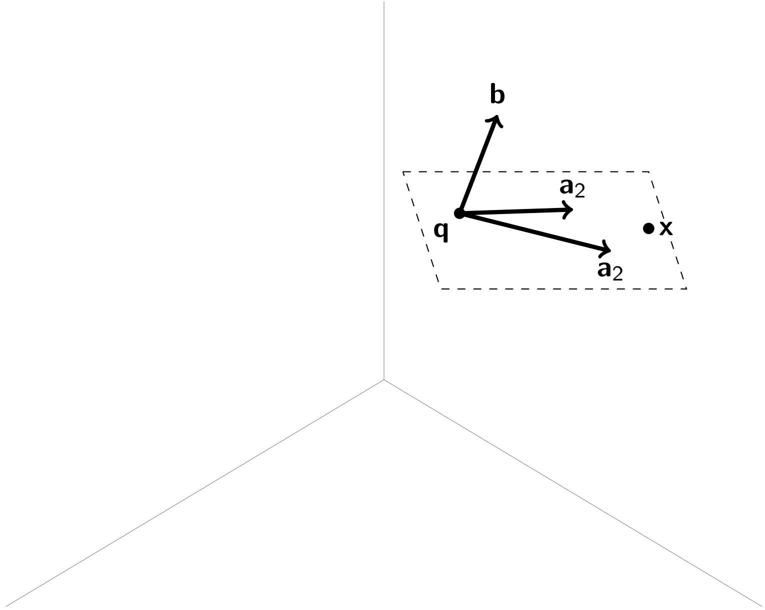

$\mathbf{a}_2$
$\mathbf{q}$
$\mathbf{x}$
$\mathbf{a}_1$

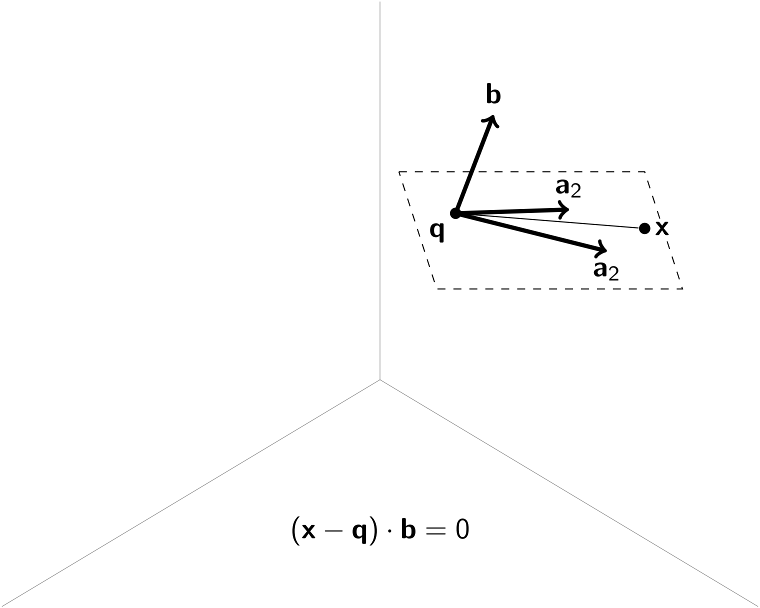


$$\boxed{\mathbf{x} = \mathbf{q} + s\mathbf{a}_1 + t\mathbf{a}_2}$$

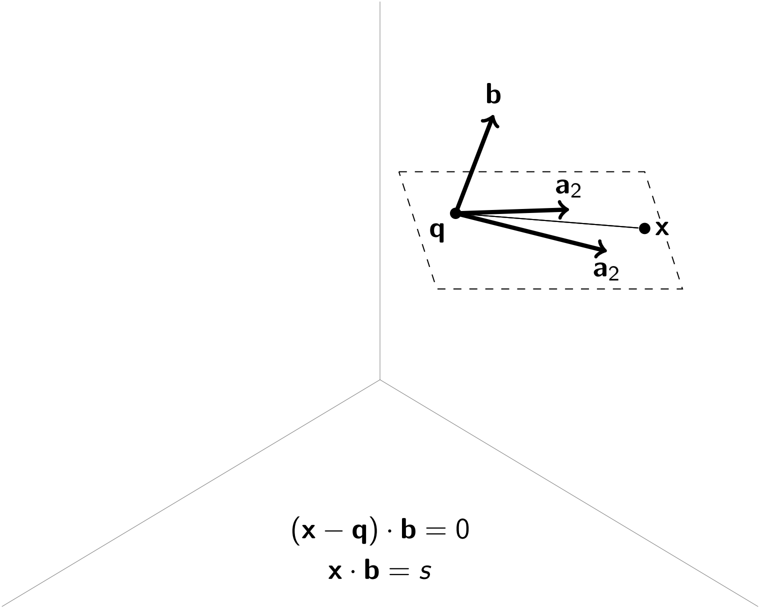

$\mathbf{a}_2$
$\mathbf{q}$
$\mathbf{a}_2$


b
$\mathbf{a}_2$
q
$\mathbf{a}_2$

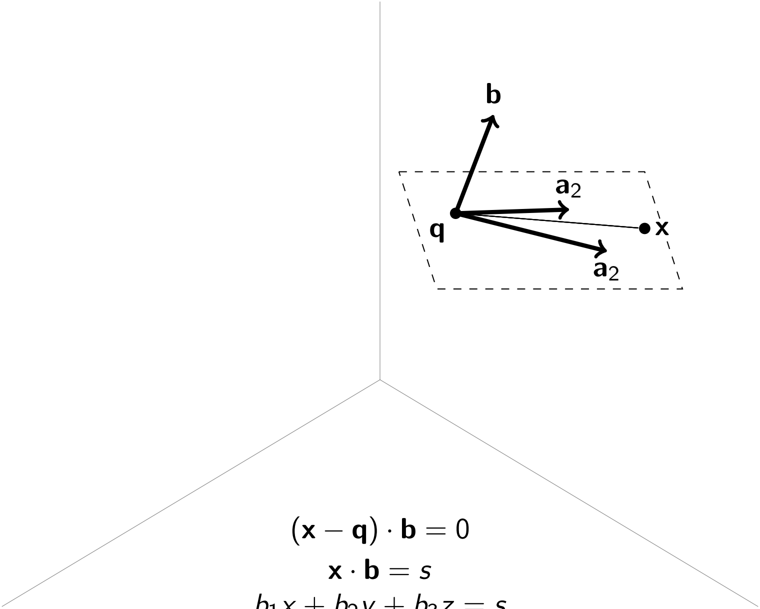

b
a_2
q
x
a_2



$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0$$



$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0$$
$$\mathbf{x} \cdot \mathbf{b} = s$$



$$(\mathbf{x} - \mathbf{q}) \cdot \mathbf{b} = 0$$
$$\mathbf{x} \cdot \mathbf{b} = s$$
$$b_1 x + b_2 y + b_3 z = s$$

Equations

	Parametric	**Component**
Line in $\mathbb{R}^2$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$b_1x + b_2y = s$
Line in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$\begin{cases} b_1x + b_2y + b_3z = s \\ c_1x + c_2y + c_3z = t \end{cases}$
Plane in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a} + t\mathbf{b}$	$b_1x + b_2y + b_3z = s$

Equations

	Parametric	**Component**
Line in $\mathbb{R}^2$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$b_1x + b_2y = s$
Line in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$\begin{cases} b_1x + b_2y + b_3z = s \\ c_1x + c_2y + c_3z = t \end{cases}$
Plane in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a} + t\mathbf{b}$	$b_1x + b_2y + b_3z = s$

Suppose $\mathbf{q}$ and $\mathbf{a}$ are vectors in $\mathbb{R}^{18}$. What would you call the geometric object resulting from the equation $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

Equations

	Parametric	**Component**
Line in $\mathbb{R}^2$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$b_1x + b_2y = s$
Line in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$\begin{cases} b_1x + b_2y + b_3z = s \\ c_1x + c_2y + c_3z = t \end{cases}$
Plane in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a} + t\mathbf{b}$	$b_1x + b_2y + b_3z = s$

Suppose $\mathbf{q}$ and $\mathbf{a}$ are vectors in $\mathbb{R}^{18}$. What would you call the geometric object resulting from the equation $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

Suppose P and Q are planes. What is the intersection of P and Q?

Equations

	Parametric	**Component**
Line in $\mathbb{R}^2$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$b_1x + b_2y = s$
Line in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$\begin{cases} b_1x + b_2y + b_3z = s \\ c_1x + c_2y + c_3z = t \end{cases}$
Plane in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a} + t\mathbf{b}$	$b_1x + b_2y + b_3z = s$

Suppose $\mathbf{q}$ and $\mathbf{a}$ are vectors in $\mathbb{R}^{18}$. What would you call the geometric object resulting from the equation $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

Suppose P and Q are planes. What is the intersection of P and Q?

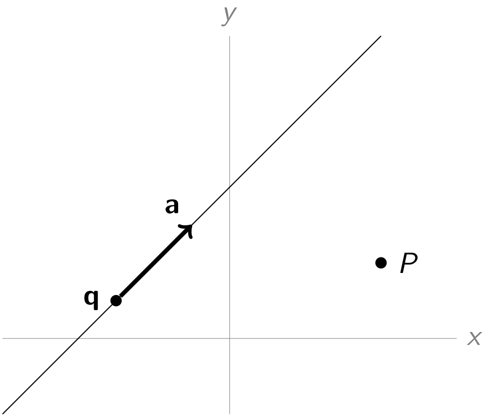
Are there any vectors $\mathbf{q}$ and $\mathbf{a}$ in $\mathbb{R}^3$ for which the equation $\mathbf{x} = \mathbf{q} + s\mathbf{a}$ is **not** a line?

Equations

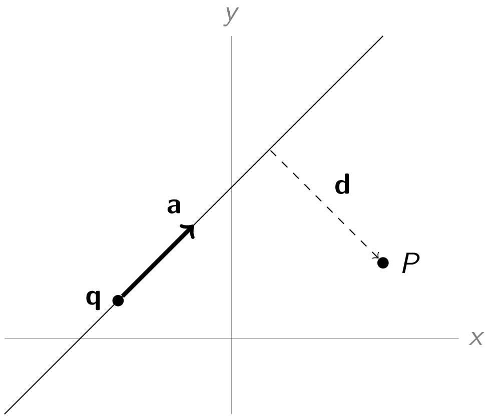
	Parametric	**Component**
Line in $\mathbb{R}^2$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$b_1x + b_2y = s$
Line in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a}$	$\begin{cases} b_1x + b_2y + b_3z = s \\ c_1x + c_2y + c_3z = t \end{cases}$
Plane in $\mathbb{R}^3$	$\mathbf{x} = \mathbf{q} + s\mathbf{a} + t\mathbf{b}$	$b_1x + b_2y + b_3z = s$

Recall: **b** was the normal vector to the plane $b_1x + b_2y + b_3z = s$.

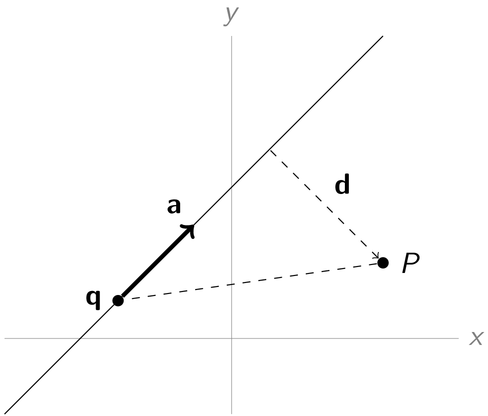
True or False: for a point P on the plane $5x + 7y + 11z = 22$, the vector with head at P and tail at the origin is orthogonal to the vector $[5, 7, 11]$.



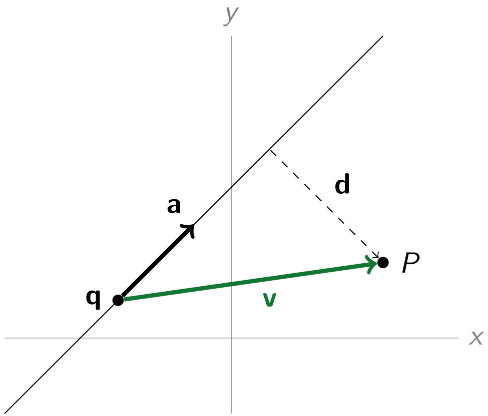
How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?



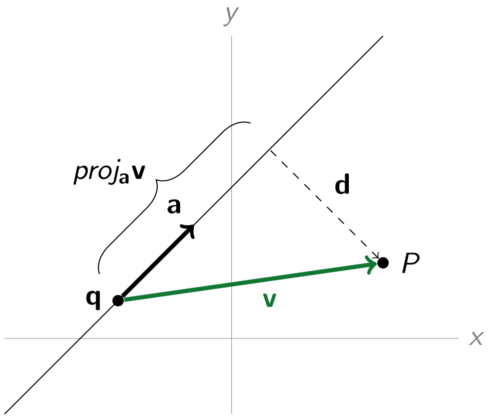
How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?



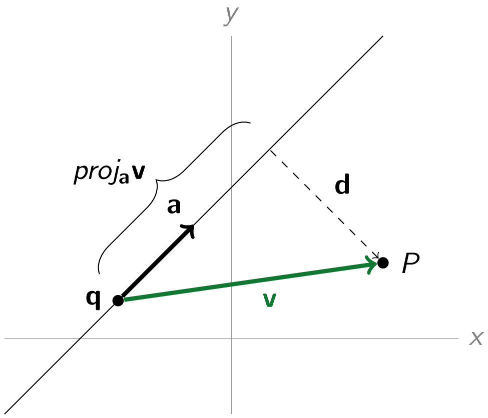
How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?



How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

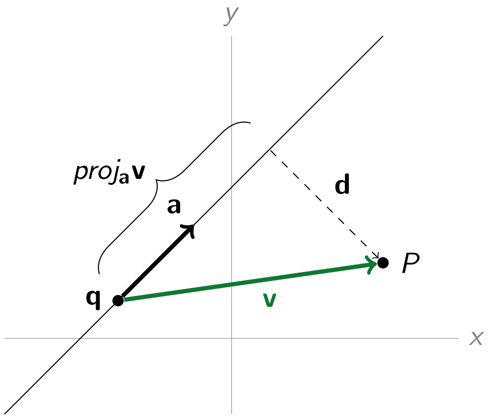


How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?



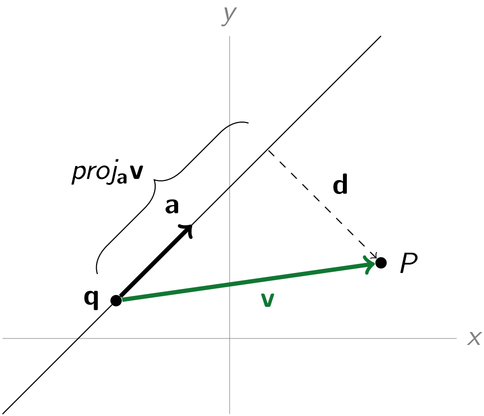
How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

$\mathbf{d} + proj_{\mathbf{a}}\mathbf{v} =$



How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

$\mathbf{d} + proj_{\mathbf{a}}\mathbf{v} = \mathbf{v}$, so



How can you find the distance from the point P to the line $\mathbf{x} = \mathbf{q} + s\mathbf{a}$?

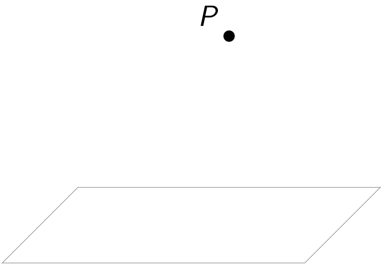
$\mathbf{d} + proj_{\mathbf{a}}\mathbf{v} = \mathbf{v}$, so $\|\mathbf{d}\| = \|\mathbf{v} - proj_{\mathbf{a}}\mathbf{v}\|$, where $\mathbf{v} = \mathbf{P} - \mathbf{q}$

Find the distance from the point $(10, 2, 3)$ to the line

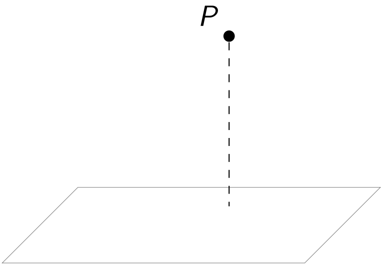
$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

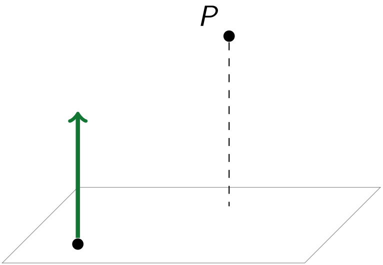


Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.



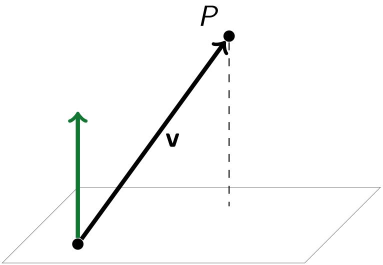
Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{n}$$



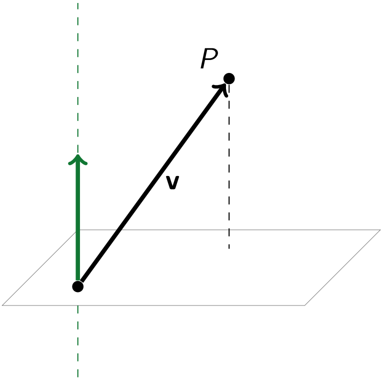
Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{n}$$



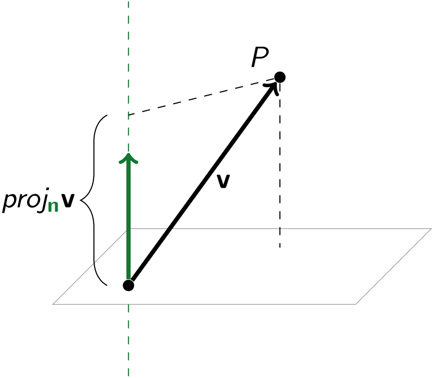
Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{n}$$

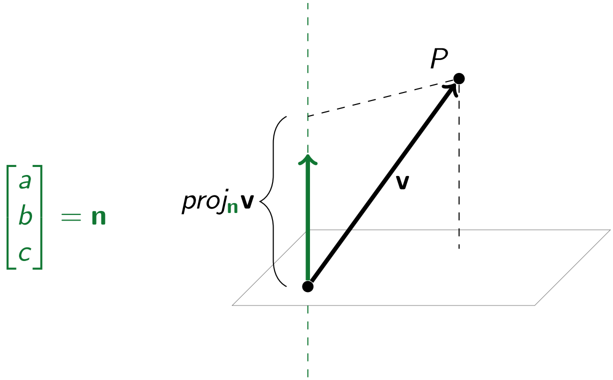


Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{n}$$



Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

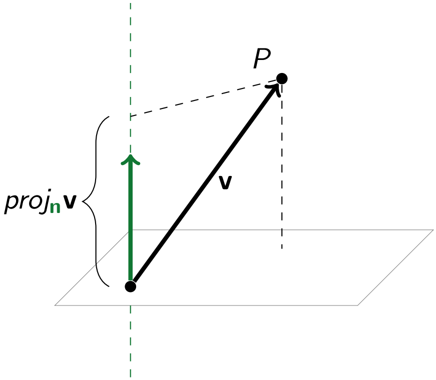


Find the distance from the point $(3, 5, 1)$ to the plane

$$\mathbf{x} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

Find the distance from the point P to the plane Q, where Q is given by the component equation $ax + by + cz = d$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \mathbf{n}$$



Find the distance from the point $(3, 5, 1)$ to the plane

$$\mathbf{x} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

$5\sqrt{26}$

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?
Nothing: they are parallel, but not identical.

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?
Nothing: they are parallel, but not identical.

Let P be the plane with equation $2x + y - z = 1$, and let Q be the plane with equation $x + 2y + 3z = 0$.

What will their intersection be: a plane, a line, a point, or nothing?

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?
Nothing: they are parallel, but not identical.

Let P be the plane with equation $2x + y - z = 1$, and let Q be the plane with equation $x + 2y + 3z = 0$.

What will their intersection be: a plane, a line, a point, or nothing?

A line: they are not parallel.

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?
Nothing: they are parallel, but not identical.

Let P be the plane with equation $2x + y - z = 1$, and let Q be the plane with equation $x + 2y + 3z = 0$.

What will their intersection be: a plane, a line, a point, or nothing?

A line: they are not parallel.

Find the intersection in parametric form.

Let P be the plane with equation $2x + 2y + 2z = 1$, and let Q be the plane with equation $x + y + z = 1$.

What will their intersection be: a plane, a line, a point, or nothing?
Nothing: they are parallel, but not identical.

Let P be the plane with equation $2x + y - z = 1$, and let Q be the plane with equation $x + 2y + 3z = 0$.

What will their intersection be: a plane, a line, a point, or nothing?

A line: they are not parallel.

Find the intersection in parametric form. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3/5 \\ -2/5 \end{bmatrix} + s \begin{bmatrix} 5 \\ -7 \\ 3 \end{bmatrix}$