Homework 1 - to be handed back on Friday, October 16

1. In this question, we assume that the yield curve is flat, i.e. the interest rate r is constant across maturities. Time t is counted in years.

   (a) Consider a zero-coupon bond with face value $100,000$ \$ and maturity T. What is its price P? The interest rate changes from r to $r + \delta r$, where δr is small, and the price of the bond then changes from P to $P + \delta P$. Find:
   $$\frac{\delta P}{P}$$
   This is called the *sensitivity* of the bond. Compute its values for $r = 3\%$, $\delta r = 0.01\%, T = 1$ and $T = 30$

   (b) Unfortunately, there are no zero-coupon bonds on that market. The only available bonds are two coupon bonds, bond S and bond L, both of which have nominal value $1,000$ \$, and yearly coupons with nominal interest 10%. The first one is exactly 1 year away from maturity, the second one is exactly 30 years away from maturity. Find their prices P_S and P_L. Find the sensitivities of these bonds. Compute their values for $r = 3\%$, $\delta r = 0.01\%, T = 1$ and $T = 30$

   (c) I want to duplicate a 10-year zero-coupon with face value $100,000$ \$ (bond Z) with the two existing bonds. Find x_1 and x_2 such that the portfolio consisting of x_1 bonds S and x_2 bonds L has the same value and the same sensitiviy as bond Z (note that x_1 and/or x_2 are allowed to be negative)

2. In this question, X is a reflexive B-space and $F : X \longrightarrow R \cup \{+\infty\}$ is a convex l.s.c function on X. Compute the following in terms of the Fenchel transform $F^* : X^* \longrightarrow R \cup \{+\infty\}$:

   (a) G^*, where $G(x) := \lambda F(x) + a$, where $\lambda > 0$ and a are constants.

   (b) G^*, where $G(x) := F(x) + < y^*, x - y >$, where $y \in X$ and $y^* \in X^*$ are given

   (c) G^*, where $G(x) := F(x) + \delta(x \mid X_0)$, where $X_0 \subset X$ is a closed linear subspace

3. Suppose $X = L^\infty$. Take a non-empty subset $A \subset X$ such that:
   $$\inf\{m \in R \mid m \in A\} > -\infty$$
   $$X \in A,\ Y \geq X \Longrightarrow Y \in A$$
   and define:
   $$\rho(X) = \inf(m \in R \mid m + X \in A)$$

   (a) Show that ρ is a monetary risk measure

   (b) Show that ρ is convex if and only if A is convex

   (c) Show that ρ is conherent if and only if A is a convex cone.