Assignment #3
To be handed in Friday, October 30

1. Let X_t be an Ito process defined by:

$$dX = \mu\left(a - \ln X\right) X dt + \sigma X dW, \quad X_0 = x$$

where μ, a, σ, x are positive constants.

   (a) Set $Y_t := \ln X_t$ and express dY_t in terms of Y_t

   (b) Find X_t

2. Consider the two Ito processes defined by:

$$\begin{aligned} dX &= \alpha_1 X dt + \sigma_1 X dW_1, \quad X_0 = x > 0 \\ dY &= \alpha_2 Y dt + \sigma_2 Y dW_2, ...Y_0 = y > 0 \end{aligned}$$

where $\alpha_1, \alpha_2, \sigma_1, \sigma_2, x, y$ are deterministic constants and W_1 and W_2 are independent BMs. Define a third Ito process by:

$$Z_t = \frac{X_t}{X_t + Y_t}$$

Express dZ in terms of Z, dt, dW_1 and dW_2

3. Let X_t be an Ito process defined by:

$$dX = \mu dt + \sigma X dW, \quad X_0 = x$$

where μ, σ, x are positive constants.

   (a) Find the expectation and the variance of X_t

   (b) Compute $d\left(Z_t X_t\right)$ where

$$Z_t = \exp\left(-\sigma W_t + \frac{1}{2}\sigma^2 t\right)$$

   (c) Express X_t in terms of s and W_s, $1 \leq s \leq t$ (the expression will involve an integral)